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A
TREATISE
OF
Opticks Direct.

SHEWING,

By New *Obſervations*, and from New *Principles*, how OBJECTS are Apprehended by the Viſive Senſe, with Reſpect to their Diſtance from the Eye, and their Oblique Situation.

Delivering alſo

Plain Rules for Drawing PICTURES againſt a Wall, to be ſeen Obliquely; which are likewiſe Applicable to the Carving STATUES, to be ſet in high Places.

To which is Added, An

APPENDIX to PERSPECTIVE:

Shewing thoſe who are verſed in that Art, the Reaſon of ſome Diſproportions in Perſpective Draughts, thro' the Ill Placing the Eye;

AND

Delivering a Rule for finding the moſt Commodious Diſtance of the Eye from the Object in that Art.

By JOHN SHUTTLEWORTH,
Rector of *Oborn* and *Lillington* in the County of *Dorſet*.

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London: Printed for *Dan. Midwinter* at the *Three Crowns*
in St. *Paul's Church-yard.* MDCCIX.

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TO THE
RIGHT HONOURABLE
THE
Earl of SUNDERLAND,
Principal Secretary of State.

May it pleaſe Your Lordſhip,

HAving for many Years convers'd with Mathematical Sciences, as I had Opportunity, I met at length with ſome Hints which put me upon ſuch Applications as have produc'd the following Papers, which I humbly crave Leave to Dedicate to Your Lordſhip;

whoſe Univerſal Character for Noble and Vertuous Accompliſhments, for Learning, and Regard to thoſe who have Pretences to it, hath made me Ambitious of your Patronage.

And beſides, I beg Your Lordſhip to be aſſured, That I have not only that profound Reſpect for Your Perſon, which in common is paid by all thoſe who are made acquainted with Your Great Character; but that I have alſo a Natural Veneration for You, as being the Head of that Family, from whom my Father and my Self have receiv'd ſo many Favours.

As to my Treatiſe, tho' I can't pretend to hope it is worthy of the Great Name I have prefix'd to it, yet I could not omit this Occaſion of ſhewing my Dutiful Regard to Your Lordſhip, and of making my ſelf known to You. And in Excuſe too, of the Imperfections of my Piece, I will venture to ſay, That whatſoever Faults it may be charg'd with, they will be allowed to be only ſuch as are imputable to want of Genius, and not ſuch as have crept in for want of Pains and Induſtry.

I humbly entreat Your Lordſhip to accept what I here offer to You as a true Token of my Gratitude to Your Family, and to caſt a favourable Eye upon my Endeavours.

And I ſeriouſly profeſs, That as it is a mighty Satisfaction to me, to ſee You Adorn'd with Great Honours, and to hear Your Eminent Vertues frequently ſounded forth; ſo I ſhall conſtantly pray to GOD to grant a Continuance and Encreaſe of his Bleſſings to Your Self and Your Noble Relations. I am

Your Lordſhip's

Moſt Obedient

Humble Servant,

John Shuttleworth.

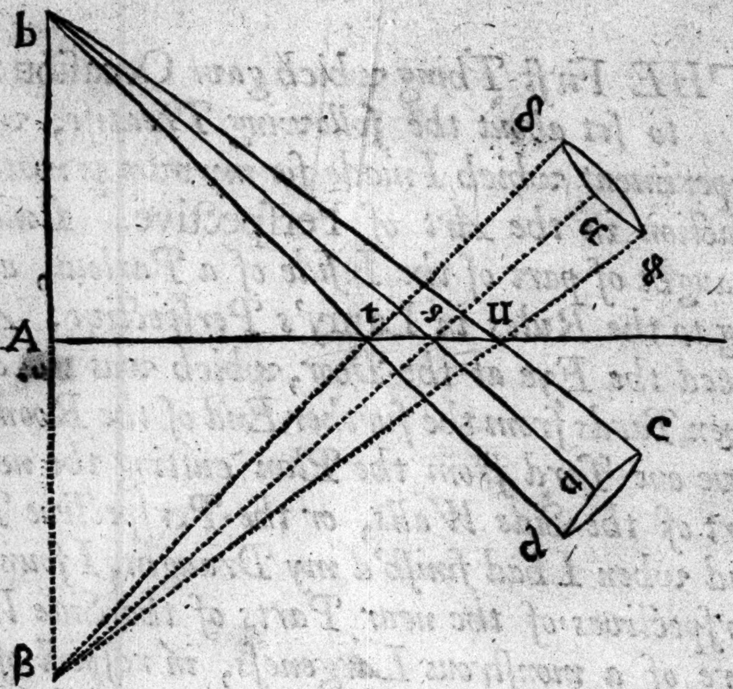
TO THE

READER.

THE Firſt Thing which gave Occaſion *to me to ſet about the following Treatiſe, was an Experiment which I made for my own private Satiſfaction in the Art of* Perſpective. *I made a Draught of part of the Inſide of a Parlour, according to the Rules in* Lamy'*s Perſpective: And I placed the Eye at the Door, which was* [redacted] *about Seven Yards from the further End of the Room,* & *not above one Yard from the Plan cutting the neareſt Part of the Side Walls, or the Perſpective Plan. And when I had finiſh'd my Draught, I found the Perſpectives of the near Parts of the Side Walls were of a monſtrous Largeneſs, in reſpect of the Perſpectives of the remote Parts of thoſe Walls. Inſomuch, that the Draught did no way tolerably reſemble the Natural Appearance of thoſe Side Walls.*

*This put me upon a Search into the Reaſon of this Difformity; and being upon this Enquiry, I found it neceſſary, in order to account for it, to find how Ob*jects *are apprehended by the Viſive Senſe; and how* *it comes to paſs that near* Objects *appear to be near, and remote* Objects *appear remote as they are.*

Having advanced ſo far, there preſently occurred to me an Hint, which laid a Foundation for me to build upon: The Occaſion *of which, and the* Application, *here follows.*



Le A b *be a ſtreight Line, and* A t s u *a Section of a plain Looking-glaſs at Right Angles to* A b.

Let b *be a Point in* A b, *emitting the Rays* b t, b s, b u, *cutting the Section of the Looking-glaſs in* t, s, u.

Produce b A *to* β, *ſo as that* A β = A b. *and draw* β t, β s, β u, *and produce them.*

Then let δ α κ *ſtand for that Part of the Eye which is capable of receiving the Viſual Rays, and occurring to the Lines* β δ, β α, β κ.

Then the Rays b t, b s, b u, *being reflected from the Looking-glaſs, are reflected into the diverging Lines* t δ, s α, u κ, *reſpectively; and thoſe Lines* δ t, α s, κ u, *do converge to* β. *And ſo it comes to paſs that the Point* b, *whoſe Rays are thus reflected from the Looking-glaſs, is ſeen as if it were in* β.

This is a Principle which the Writers of Opticks *go upon; particularly it may be ſeen in Dr.* Gregory's Catoptricks.

And this doth ſuppoſe, that if the Looking-glaſs were taken away, and the Eye placed at d a c, *occurring to the Rays* b c, b a, b d, *then would the Point* b *be ſeen where it is, namely, in the common Interſection of the Lines* d b, a b, c b. *which is called a* Focus.

The Conſideration of this gave me a Hint, that I might determine the ſeeming Diſtance of the Points of an Object *by this Principle; and accordingly I make uſe of it in the* 5th Poſition, p. 2.

Then I improve upon this, by conſidering, that the Point b, *in the Line* A b, *which is thus ſeen, is not an Indiviſible Point, or* quid minimum; *becauſe it appears, by* Obſervation, *that an* Object *of ſome Bigneſs will not be apprehended at a great Diſtance, tho' it hath all the Advantages which* Colour *can give it; and the Bigger any* Object *is, the farther is it capable of being diſcerned by the Eye. I concluded therefore, that the Point* b *muſt be of ſome Bigneſs; and ſtill the farther the Eye is diſtant from* A b, *the bigger muſt the Point* b *be. So that it appeared very natural to me, to conceive the Eye capable only of apprehending a Body of Rays of ſome determinate Magnitude, or under ſome determinate Angle; which Body of Rays forms a Cone, whoſe Vertex is in the Eye, and whoſe Baſis is the Point ſeen: And ſo I conceived that Multitudes of ſuch Bodies of Rays being emitted, namely, one to each ſeveral Point of that Part of the Eye which is capable of receiving the Rays; and an Interſection being made by theſe Cones proceeding athwart to the Extremities of the Portion of the Eye which apprehends them; I ſay, I concieved that every Point was ſeen in ſuch Interſection, even as I have more fully expreſſed it in the* Second Obſervation, pag. 8. *And this I pitch upon as the Fundamental Principle of my Treatiſe, joining thereto an* Obſervation *concerning the different Appearance of the Length of Lines, in reſpect of their oblique Situation; which you have in the* Firſt Obſervation, pag. 4. *and which I found neceſſary to make, becauſe tho' each Point* *of a Line* A b, *be ſeen in the Place it ſelf, where the Point is, (as it muſt needs be in* Objects *which are very near) yet if* A b *be turned upon its Center* A, *ſo as to be oblique to the Eye* d a c; *the more oblique it is, the leſs Length will the Line* A b *ſeem to have.*

Upon theſe Principles I proceed by Algebraic Reaſoning and Geometrical Operations. But at length I come to ſuch Rules for drawing Pictures to be ſeen obliquely againſt a Wall, carving Statues to be ſet in high Places, and finding the moſt commodious Diſtance of the Eye *from the* Object *in the Art of* Perſpective, *that any one who only underſtands* Euclid's Geometry, *and* Trigonometrical Calculation, *may make uſe of them*

He which works by theſe Rules, would do well to uſe Neper's Bones *in his Calculations; becauſe the Numbers he will have to deal with, are too big for any Tables of* Logarithms: *And likewiſe he ſhould have a Canon of Natural* Sines *and* Tangents, *which expreſſeth not only Minutes, but Seconds.*

And in making uſe of the Segmental, *according to the Method preſcribed in the* Firſt and Second Lemma's, *and* Firſt and Second Scholiums, *foregoing the* Tenth Problem; *and alſo in dividing the Line* e m, &c. *by the Preſcription of the* Ninth and Tenth Problems, *becauſe theſe Things are much done by Geometrical Delinea-*

tions, and conſequently cannot be performed very nicely; it would not be amiſs to govern theſe Operations *by conſidering that* $ka < ab$, *and* $ab < bc$, &c. *in the* Lemma's *and* Scholiums *foregoing the* Tenth Problem; *and alſo, that* $eA < AB$, *and* $AB \leqq BC$, &c. *in the* Ninth and Tenth Problems.

As for the Obſervations *I uſe, they are ſuch as I could make with my bare Eye, abſtracting a Judgment of Reaſon as much as I could; but if* Inſtruments *could be deviſed to be applied to the* Obſervations, *I make no doubt but that the* Rules *I give might be more exact. In the mean Time, if what I have offer'd may afford an Hint to Abler Heads, to put them upon introducing a better Syſtem of the Matters contained in this Treatiſe, I ſhall be heartily glad to have been the* Occaſion *of it, and ſhall think I have not failed of my* End.

A
TABLE
OF
CONTENTS.

Page.

I. *Definition of the Eye.* 3d Poſition. 1.

II. *Why One Object, ſeen by the Two Eyes, appears but One.* 1ſt Cor. *of the* 5th Poſition. 3.

III. *Why one Object more remote from the Eye than another, appears more remote.* 2d Cor. *of the* 5th Poſition. ibid.

IV. *The*

Page.

IV. *The Deſcription and Uſe of the Segmental.* Obſerv. 1. Prob. 1. 2d Lemma. 1ſt *and* 2d Scholiums *foregoing the* 10th Problem. 5, 6, 50, 51, 54.

V. *The New Principle, ſhewing the Ground for finding the ſeeming Diſtance of remote Objects.* Obſerv. 2. 7, 8.

VI. *The real Diſtance of an Object being given, to find the ſeeming Diſtance.* 2d Probl. 11.

VII. *A ſtreight Line at a Diſtance appears curve concave toward the Eye: Together with the Reaſon why the remote Parts of Two parallel Rows of Trees, or of Two parallel Walls, ſeem to approach nearer and nearer to each other to a Perſon in the midſt of them.* Cor. *of the* 1ſt Theorem 15.

VIII. *The Diſtance from the Eye, and Poſition of a ſtreight Line of an infinite Length being given, to draw the Curve which ſhall exhibit the ſeeming Appearance of ſuch ſtreight Line.* Prob. 3. 17, 18.

IX. *The ſeeming Diſtance from the Eye of an Object being given, to find the real Diſtance of it.* Problem. 4. 21.

X. *A Line which muſt appear ſtreight, muſt be a Curve convex toward the Eye.* Cor. *of the* 2d Theorem. 24.

XI. *The ſeeming Diſtance of a ſtreight Line being given, to draw the Curve which ſhall exhibit ſuch ſtreight Line.* Problem 5. 28.

XII. *The*

Page.

XII. *The moſt remote Object doth not appear to be diſtant from the Eye above 2500 Inches.* Cor. *of the* 4th Obſerv. 33.

XIII. *The Reaſon why the Hills or Plains which bound our Sight, being at Three Miles diſtance, or upward, appear round.* Cor. *of the* 3d Schol. *of the* 4th Obſerv. 35.

XIV. *The real Diſtance, Poſition, and Length of a Line being given, to find the ſeeming Length of ſuch Line.* Problem 6. ibid.

XV. *The ſeeming Diſtance, Poſition, and Length of a Line being given, to find the real Length of ſuch Line.* Problem 7. 38.

XVI. *To divide a Line ſo, as that each Diviſion of it ſhall appear equal to one Inch in Length:* Probl. 9. *and* 10. 44, 57.

XVII. *To conſtruct a Figure againſt a Wall or a Frame divided into leſſer Figures, which ſhall exhibit to the Eye a chequered Parallelogram divided into little Parallelograms, whoſe Sides ſhall appear equal to One Inch in Length.* Probl. 11. 59, 60, &c.

XVIII. *The manner of applying this to the drawing Pictures againſt a Wall to be ſeen obliquely, and to the carving Statues to be ſet in high Places.* Cor. *of the* 11th Problem. 63.

XIX. *The Application of a Particular Caſe.* 64.

In

In the *Appendix* to *Perſpective.*

	Page.
XX. *The Reaſon why the Perſpectives of the neareſt Parts of Objects perpendicular to the Picture or Perſpective Plan, exceed in Breadth the Perſpectives of the remote Parts of thoſe Objects diſproportionably when the Eye is placed near the Object.* Cor. *of the* 2d *Paragraph of* Schol.	80
XXI. *A Rule for finding the neareſt commodious Diſtance for placing the Eye in the Art of Perſpective.*	*penult.*

ERRATA.

Page 2. laſt Line, put *e b.* inſtead of *c b.*
Pag. 14. Line 3. put *I b.* inſtead of *I.*
Pag. 16. below the laſt Line, blot out *Draw*, and put in *Problem.*
In the Fig. Pag. 64. mark the Point of Concurrence of the Lines *m e* produced, and the Line proceeding from *l*, almoſt parallel to *e s*, with the Letter *g*.

A
TREATISE
OF
Opticks Direct.

POSITION I.

A Ray is a Line proceeding from a Point of an Object, ſo qualified, as by the mediation of the Eye to exhibit the Appearance of that point to the viſive Senſe.

POSITION II.

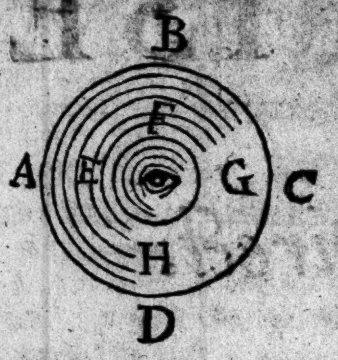
From every point of an Object innumerable ſuch Rays do proceed Orbicularly, except where hindred by Solid Bodies contiguous to that point; ſo that every point of an Object in reſpect of its Rays, is as a Center in reſpect of a Segment of a Globe.

POSITION III.

The Eye is an Inſtrument whoſe external Superficies appearing is globoſe and voluble at Will, from the direct horizontal poſition of the Axis of the Pupilla, every way round its Center, within certain Limits: And it is naturally ſo contrived by the Author of all Things, as that the Rays of every Point of an Object, either perpendicularly or obliquely (within certain Limits) darting againſt a particular Portion of this external Superficies, are tranſmitted ſo as that the Figure and Colour (except in ſome Caſes of Refraction) of ſuch Object is repreſented clearly to the Viſive Senſe.

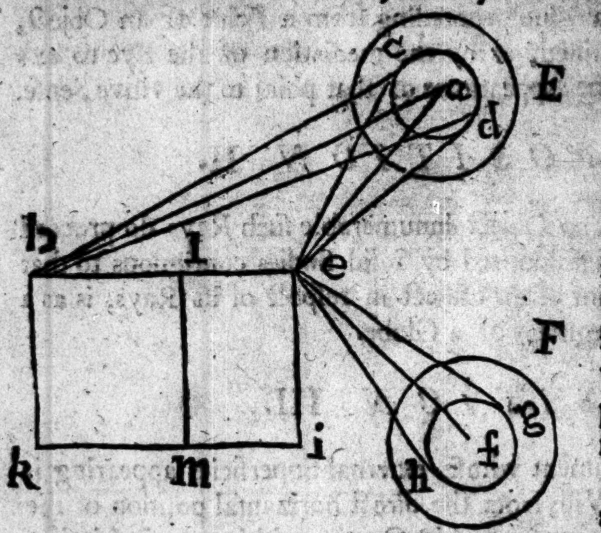
POSITION IV.

That part of the External Superficies of the Eye which tranſmits the Rays, is a ſmall Orbicular Portion of it; the Circle of which Portion terminating it, I ſhall call its Baſis; and the part tranſmitting the Rays, I call the Orbicular Portion; the Area of the Circle *A B C D*, ſtands for the whole Eye; the Area of the Circle *E F G H*, ſtands for the Orbicular Portion; and the Circle *E F G H*, is the Baſis of the Portion.



POSITION V.

The Rays of every point of an Object, with the Baſis of the Orbicular Portion form a Cone; and by the direction of the Rays of a Point from the Baſis *E F G H*, the place of that Point is determined. The Point of Interſection of thoſe Rays proceeding backward as it were from the Baſis, being taken for its place by the Viſive Senſe. It being otherwiſe unconceivable how Objects appear more and leſs diſtant as they are.



Thus the Points *e*, and *b*, in the Object *e b* are ſeen in *e* and *b* reſpectively; becauſe by the Direction of the

Rays *a e*, *b e*, *d e*, they interſect each other in *e*; ſo by the directi- of the Rays *a h*, *c h*, *d h*, they interſect each other in *h*; and the like may be ſaid of every other Point in *e h*.

COROLLARIES.

I. Hence the Object *e i*, ſeen by the Two Eyes *E*, *F*, is ſeen as one Object by them both; becauſe by the direction of the Rays *a e*, *b e*, *d e*, and of *f e*, *g e*, *h e*, to the Two Eyes ſeverally, their interſection is in the ſame Point *e*; therefore the Point *e* is ſeen at the ſame Place, and conſequently as one Point by the Two Eyes; and ſince the like may [illegible]id of every other Point in *e i*, therefore the whole Object is ſe[illegible]e ſame Place, and conſequently as One Object by both the Eyes *E*, *F*.

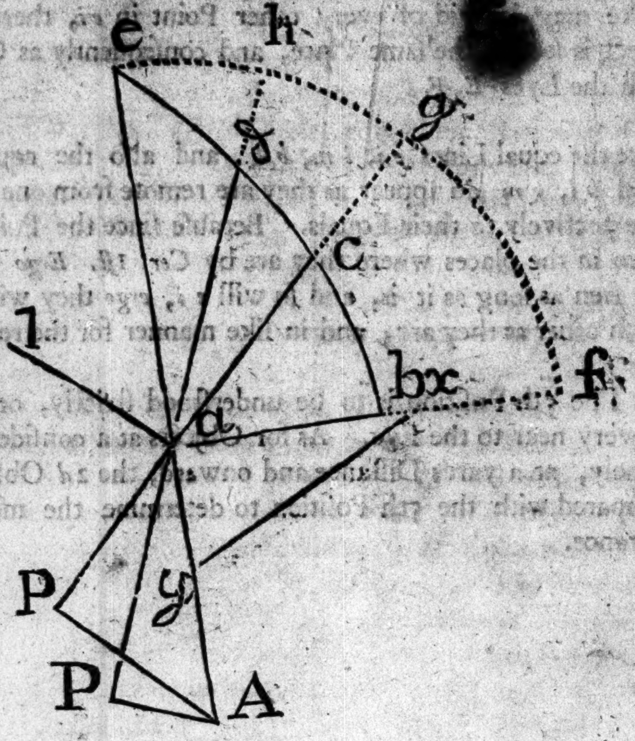
II. Hence the equal Lines *e i*, *l m*, *b k*, and alſo the equal Lines *e l*, *m i*, and *b l*, *k m*, do appear as they are remote from one another, and equal reſpectively to their Equals. Becauſe ſince the Points *b*, *k*, *e*, *i*, are ſeen in the places where they are by *Cor. 1ſt.* *Ergo* the Line *b k*, will be ſeen as long as it is, and ſo will *e i*, *ergo* they will appear equal to each other as they are; and in like manner for the reſt.

1. *Note*, The 5th Poſition is to be underſtood ſtrictly, only when Objects are very near to the Eye. As for Objects at a conſiderable diſtance, namely, at a yards Diſtance and onward, the 2*d* Obſervation muſt be compared with the 5th Poſition to determine the manner of their appearance.

2. *Note*, The Second Corollary, even when Objects are moſt near, is to be modified by the following Obſervation; by reaſon that a Line being ſeen obliquely by the Eye, will appear ſhorter than it will when it more nearly at Right Angles croſſeth the Sight at any diſtance.

OBSERVATION I.

Let the Eye be at *A*, and at the Point *a*, diſtant Three Inches from *A*, draw *af* at Right Angles with *Aae*, and in the horizontal Plane *Aabe*. Let *ab* = one Inch, and erect *al* = *ab*, perpendicular to the Plane *abe*; draw *acg*, making the Angle *gaf* = 45° : Then will *ac* = one Inch + $\frac{1}{10}$, appear = *al* = *ab*, draw *adb*, making the Angle *baf* = 67°.—30′. Then will *ad* = one Inch + $\frac{2}{10}$, appear = *al* = *ab*. Alſo *ae* = one Inch + $\frac{2}{10}$, appears = *al* = *ab*, as by the proceſs of the Curve above *d*, I gather. So a Curve being drawn through the Points *b*, *c*, *d*, *e*, and the Figure *abcde*, being placed horizontally of the height of the Eye, and in the



the Situation deſcribed, the Figure will appear like a Quadrant of a Circle whoſe Radius equal one Inch.

This Figure *a b c d e*, (which we will call a Segmental) being cut out in Braſs or Tin, muſt be had for Uſe. This Segmental is appropriated to my Eye, and to the Eyes of all thoſe who ſee as I do. As for old Men, or others who ſee otherwiſe, a Segmental may be had proper for them, by the ſame Method which I have uſed for finding this. The Lines *P a c*, *P a d*, are one Line drawn horizontally upon a Slate placed againſt the ſide of a Box, and the Lines *P A*, *P A*, are parts of the other ſide of the Box, whoſe Angle *A P a*, is a Right Angle, and the Eye at *A* ſees through a little Hole.

1. *Note*, The Parts of *a b*, near *b*, (being ſeen more obliquely than Parts of equal Magnitude near *a*) will be ſeen leſs than thoſe in ſtrictneſs; and conſequently the whole Line *a b*, being no bigger than an Inch, will appear ſomething leſs than an Inch. But here the obliquity is but ſmall; and beſides, the Number of Parts diſtinctly ſeen is ſo few, that though according to the Natural Order of this Method, for the Reaſon given, the appearing Magnitude of *a b* ſhould be leſs than its real Magnitude, yet will the difference be inſenſible.

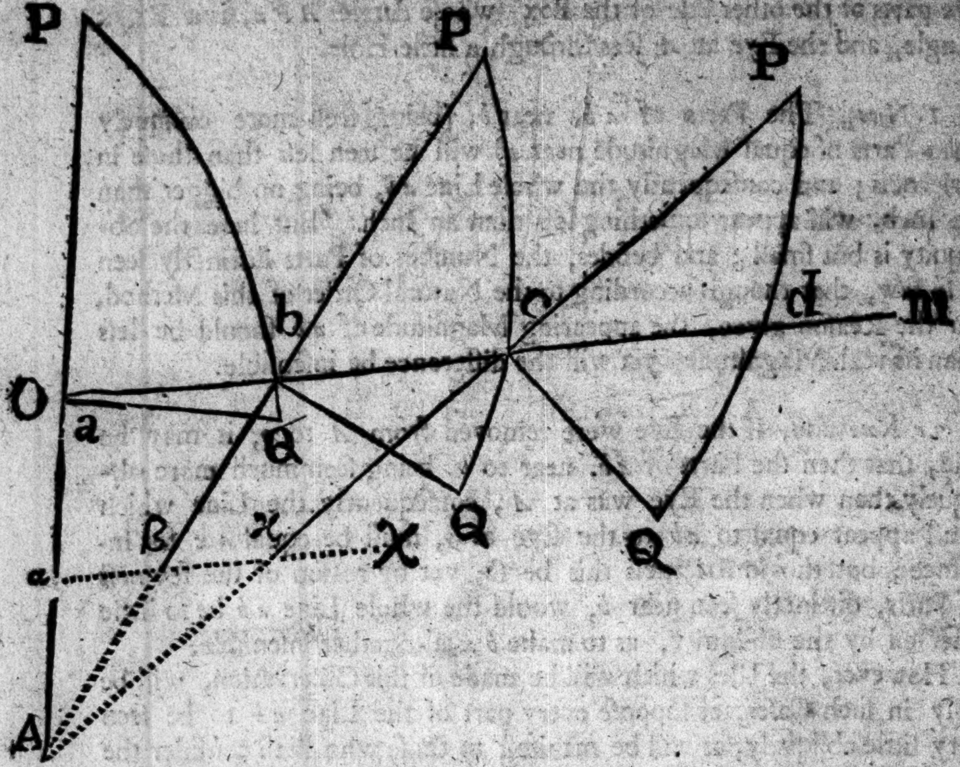
2. *Note* alſo, If the Eye were removed from *A* to *y*, it may be ſaid, that then the Parts of *a b*, near to *b*, being ſeen much more obliquely than when the Eye was at *A*; conſequently the Line which muſt appear equal to *a b* to the Eye at *y*, muſt be equal *a x* for Inſtance; but tho' in ſtrictneſs this be ſo, yet by reaſon of the fewneſs of Parts, diſtinctly ſeen near *b*, would the whole Line *a b* be ſo little affected by the obliquity, as to make *b x* altogether inſenſible.

However, the Uſes which will be made of this Obſervation, will be only in ſuch Caſes, as ſuppoſe every part of the Line *a b* to be ſeen very little obliquely, as will be manifeſt to thoſe who ſhall conſider the Uſes I make of it.

Problem

Problem I.

Let the Line *a m*, of a given or of an Indefinite length, be expos'd to Sight, the Eye being at *A*; and let it be required to divide it into Parts, *viz.* *a b*, *b c*, *c d*, each of which Parts ſhall appear equal to One Inch; working in conſequence of the First Obſervation, without any regard had to the Diſtance of *a* from *A*.



Draw the ſtreight Line *A a P*, and at the Point *a* apply the rectangular point of the Segmental, *viz.* O, applying alſo its ſide O P upwards to the Line *A P*, as in the Scheme, and drawing by it the Fig. O P Q, you have *b a*, appearing equal to one Inch.

Then drawing *A b P*, and applying the Point O of the Segmental to the Point *b*, applying alſo its ſide O P, upwards to the Line *A b P*, as in the Scheme, and drawing the Figure O P Q, you have *c b* appearing equal to an Inch.

N.B. The points to describe the Conchoid when Om is perpendicular to AO

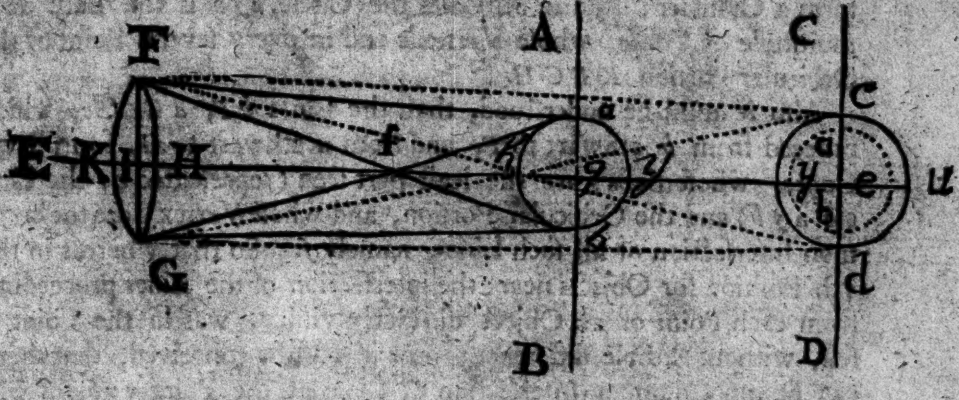
And ſo repeating ſeverally the Application, you have $d\,e$, &c. The reaſon of this Proceſs is ſo obvious, that it needs no Proof.

1. This *Problem* will be of great Uſe hereafter, and will be applicable to Caſes where the Eye at A, will be conſiderably diſtant from the neareſt part of $a\,m$; and therefore becauſe in ſuch Caſe it will be impracticable to draw the whole length of $A\,a\,P$, $A\,b\,P$, &c. you may take ſome aliquot part of $a\,A$, *viz* $A\,\alpha = \frac{1}{2}$, or $= \frac{1}{3}$, or $= \frac{1}{10}$, &c. of $a\,A$; and drawing $\alpha\,\chi$ parallel to $a\,m$, take out of it $\alpha\,\beta$, in ſuch ſort as that $a\,A : A\,\alpha :: a\,b : \alpha\,\beta$; and then drawing $\beta\,b\,P$, this will be a part of $A\,b\,P$, by the Second Prop. of the Sixth Element. (*See the Second Schol. foregoing*, Problem 10.)

And then having $b\,c$ by the Segmental as before, proceed to take $\beta\,\kappa$, ſo as that $a\,A : \alpha\,A :: b\,c : \beta\,\kappa$; and ſo proceed for all the Diviſions.

2. If your Line $a\,m$, be too long to be drawn upon your Table, you may begin with a part of it. For Inſtance, with $a\,c$, and then begin again with the remaining part $c\,m$, keeping to the Angles $\kappa\,c\,m$, $c\,\kappa\,\chi$. (*See the Second Schol. foregoing*, Problem 10.)

Let E repreſent the Center of the Eye, and $F\,H\,G\,K$, the Orbicular Portion. Let $A\,B$, $C\,D$, two parallel Lines be expos'd to Sight.



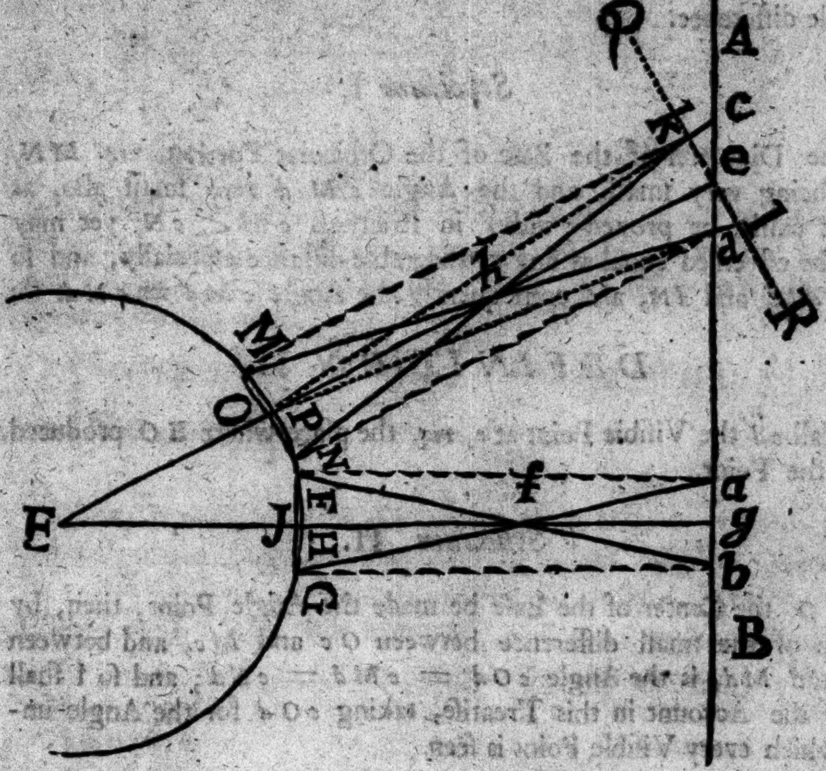
OBSERVATION II.

The Eye ſees not ſo nicely as to perceive the ſmalleſt Magnitude capable of diffuſing Rays, for a ſmall Magnitude, for Inſtance, the Circle $a\,y\,b$, which appears clearly at the diſtance of $I\,g$, from the Eye; if it be removed farther by the length of $g\,e$, it will not be apprehended there: But if $a\,y\,b$ there, be enlarged to $c\,u\,d$, it will then be clearly ſeen. So that every leaſt part that is diſtinctly ſeen, ſeems to be the Baſis of a multitude of Cones, whoſe Vertex's end in every ſeveral Point of the Orbicular Portion: And this Baſis of theſe multitudes of Cones ſeems to be ſo big, as to diffuſe Rays to F, G, the Extremities of the Baſis of the Orbicular Portion, in ſuch a manner as that the Angles *a F b*, *a G b*, *c F d*, *c G d*, are each equal to one another; a body of Rays ſeen under the Angle *a F b*, appearing but as one ſingle Ray: So that whereas it is ſaid in the 5th Poſition, that the Orbicular Portion was the Baſis of a Cone, and each Point of an Object the Vertex of ſuch Cone (which according to the Firſt Note following the Corollaries there is to be underſtood, only when Objects are ſo near as that the leaſt Parts which are viſible, and which are ſeen under the Angle *a F b*, are ſo ſmall, as to be of no conſiderable bigneſs.) Now it muſt be underſtood, that for Objects at a Diſtance, the Orbicular Portion is either the Baſis of a Curti-Cone, formed by the Lines *a F*, *b G*, &c. and by the Circle *a y b*, *viz.* When *a y b*, tho' at a diſtance, is yet ſo near as to be leſs than the Orbicular Portion; or elſe the Circle *a y b* is the Baſis of a Curti-Cone, formed by the Lines *a F*, *b G*, &c. and the Orbicular Portion, *viz.* when *a y b* is ſo far off as to exceed the Orbicular Portion; but when the Orbicular Portion, and *a y b* are equal, then thoſe Circles, with the Lines *a F*, *b G*, &c. form a Cylinder: And in all Caſes the Circle *a y b* is the Baſis of a multitude of Cones, whoſe Vertex's end in every ſeveral Point of the Orbicular Portion *K F G H*.

Alſo it muſt be underſtood, that the Lines *b F*, *a G*, &c. which proceed from *b* athwart to F, and from *a* athwart to G, form Two Cones, whoſe Baſis's are the leaſt part ſeen (repreſented by the Circle *a y b*) and the Orbicular Portion, and whoſe Vertex's are the common Interſection of all ſuch Lines, namely *f*. So that whereas in the 5th Poſition for Objects near; the interſection of the Lines proceeding from each Point of an Object diſtinctly viſible, was in the Point it ſelf, without ſenſible difference; now it muſt be conceived, that there can be no Common Interſection of Lines at the Point *a y b*, or between the Eye and the leaſt Point viſible, only in *f*; and ſince the Bodies of Rays ſeen under the Angles *a F b*, *a G b*, are ſeen but as ſingle Rays, therefore for the Reaſons in the 5th Poſition, I ſhall (in this Treatiſe) take *f* the Common Interſection of the Lines *a G*, *b F*, &c. for the place where *a y b*, the leaſt Point viſible is ſeen. There being many Obſervations to be brought with which this Aſſumption agrees, and nothing in Reaſon or Experience to contradict it.

DEFINITION I.

I call the leaſt Parts of an Object which are diſtinctly ſeen, Viſible Points.



Let *E* be the Center of the Eye, and *MPN*, *FHG*, ſtand each for the Orbicular Portion; and *O* and *I* the Center of the Baſe of that Portion.

Let the Line *AB* be expoſed to Sight, and let *ced*, *agb*, be Viſible Points in that Line.

Draw *EO* and *EI*, paſſing thro' *O* and *I*, the Centers of the Baſe and produce them.

OBSERVATION III.

The Orbicular Portion is rowled againſt every Point of an Object ſucceſſively, ſo as that the Lines EIg EOe, paſs through O and I, at Right Angles to MN, FG, and cut the Points reſpectively; and the place where the Point is ſeen, is in the Line Ee, Eg, without ſenſible difference.

Scholium I.

The Diameter of the Baſe of the Orbicular Portion, *viz.* MN, FG, being very ſmall, and the Angle cMd very ſmall alſo, as might eaſily be proved; altho' in ſtrictneſs $cM < cN$, yet may they be eſteemed equal at any conſiderable diſtance eſpecially, and ſo may dM, and dN, and conſequently the Angle $cMd = cNd$.

DEFINITION II.

I call cd the Viſible Point at e, *viz.* the place where EO produced cuts the Point.

Scholium II.

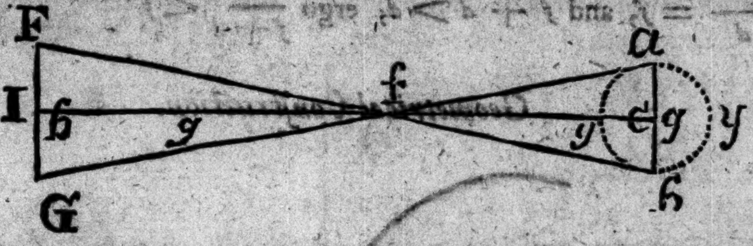
If O, the Center of the Baſe be made the Angle Point, then, by reaſon of the ſmall difference between Oc and Mc, and between Od and Md, is the Angle $cOd = cMd = cNd$; and ſo I ſhall make the Account in this Treatiſe, taking cOd for the Angle under which every Viſible Point is ſeen.

Scholium III.

If the Line QR paſs thro' e, and parallel to MN, then would the Viſible Point at e, in QR, be ſeen at the ſame place where the Viſible Point at e, in AB is ſeen, without the leaſt ſenſible difference; and ſo I ſhall make the Account in this Treatiſe, for the conveniency of Calculation, taking the Viſible Point kel in QR, for ced in AB, and taking the Angle kOl for cOd; alſo conſidering Oe, as biſecting the Angle kOl, and taking b, the Point of Interſection of lM with kN, &c. for the place at which the Viſible Point cd is ſeen, or for the Point of Interſection of dM with cN, &c.

Problem II.

The Diſtance of the Viſible Point at *g*, being given from the Eye at *I*, to find the Diſtance of *f* from the Eye, *viz*. The Point at which



the Viſible Point *a y b* is ſeen. Call the Diameter of the Viſible Point at $g = c$; and call $Ig = d$; and call $FG = b$; and $If = g$; and $fg = y$ *F G*, *a b*, *Ig*, &c. being ſuppoſed as in the *Third Obſervation* and *Scholiums*.

1 | $b : c :: g : y$, by *Prop.* IV. the 6th *Elem.* ergo $b + c : b :: g + y\ (d) : g$.

2 | Ergo $g = \frac{b\,d}{b + c}$.

Now, becauſe let *d* be of what length it will, *c* will ſtill be the Subtenſe of the Angle *a I b*; by *Obſ.* 2. and 3*d.* Schol. of *Obſ.* 3. And ſo *c* will ſtill have the ſame proportion to *d*, be *d*, ever ſo long or ever ſo ſhort. Therefore if we call

3 | $c = \frac{d}{l}$ *l* will be a ſtanding Quantity the ſame in all Poſitions of *d*.

Then changing *c* in the Second Step to $\frac{d}{l}$

4 | $g = \frac{b\,d}{1} \div \frac{b\,l + d}{l} = \frac{b\,d\,l}{b\,l + d}$.

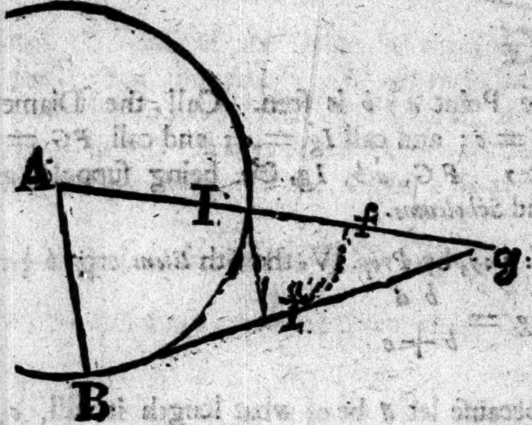
But *b* is alſo a ſtanding Quantity, being the Diameter of the Baſe of the Circular Portion; and though the Baſe may be thought to contract it ſelf a little upon ſome Occaſions, yet generally for Objects at a Diſtance, it keeps to one Magnitude; and ſo *b* may be conſidered as a ſtanding Quantity too; and conſequently *b l* is a ſtanding Quantity. Call $b\,l = f$, and change the 4th Step accordingly, ergo

5 | $g = \frac{f\,d}{f + d}$

COROLLARY.

Hence g will always be leſs than f, be d ever ſo long, for becauſe $\frac{fd}{d} = f$, and $f + d > d$, ergo $\frac{fd}{f+d} < f$.

Geometrical Conſtruction.



Be f what it will, it may be aſſumed at preſent $= AI$.

Then with the Radius AI, deſcribe the Circle ABI, and producing AI indefinitely, take $Ig = d$.

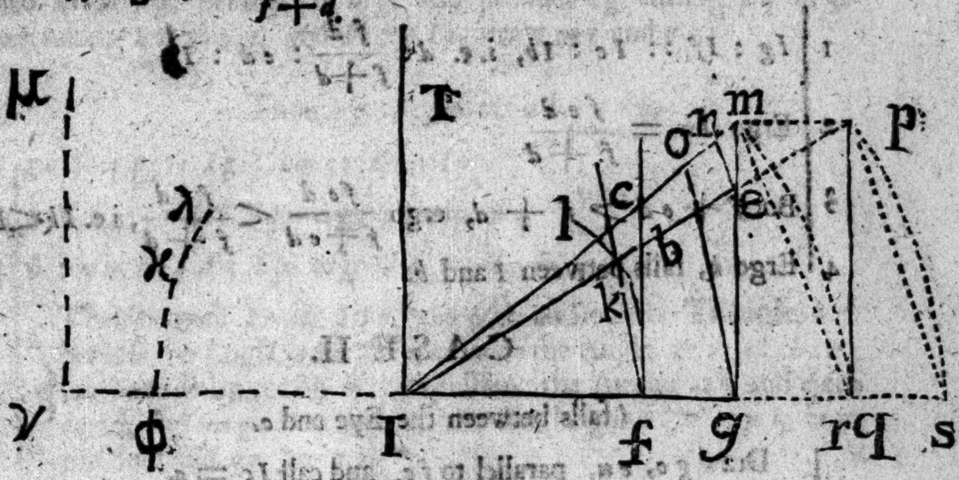
Then drawing the Radius AB any how, draw Bg; and from I, draw If parallel to AB, cutting Bg in f.

Then will If be the required Quantity, and may be taken out of Ig, as in the Scheme.

Demonſtration.

Demonſtration.

For $Ag = f + d$, and $AB = f$, and $Ig = d$.
But $Ag : AB :: Ig : If$, by *Prop.* IV. 6th Elem. *i. e.* $f + d : f :: d : If$, ergo $If = \frac{fd}{f+d}$.



Theorem I.

Let the Line meg be expoſed to Sight, the Eye being at I, and the Angle Igm not leſs than a Right Angle, and let the Viſible Points at m, e, g, be ſeen in l, k, f.

Draw fb parallel to mg, cutting Ike in b, and draw fke, cutting Ilm in c.

Then will k fall between the Eye and b; and alſo l will fall between the Eye and o.

Note, By reaſon of the ſmall length of EI, EO, &c. (*See* Fig. *for* Obſ. III.) If I be conceived to coincide with E, and ſo the Angle OEI to vaniſh; (FG ſtill being ſuppoſed to be at Right Angles with Ig, Ie, Im, &c.) the Difference in Account for Objects at a Diſtance, will be inconſiderable. And ſo for the eaſe of Calculation, I ſhall make the Account in this Treatiſe.

Demonſtration.

$$\text{Call}\left\{\begin{array}{l} Ig = d \\ Ie = ed \\ Im = id \end{array}\right\}\begin{array}{c}\text{Ergo} \\ e > 1 \\ i > e\end{array}\left\}\begin{array}{l}\text{and then by} \\ \text{Problem II.}\end{array}\right\{\begin{array}{l} If = fd \div f + \\ Ik = fed \div f + \\ Il = fid \div f + \end{array}$$

CASE

CASE I.

(*See the foregoing Figure*)

k will fall between *b* and *I*.

1 $Ig : If :: Ie : Ib$, i.e. $d : \frac{fd}{f+d} : ed : Ib$

2 Ergo $Ib = \frac{fed}{f+d}$

3 But $f + ed > f + d$, ergo $\frac{fed}{f+ed} \leq \frac{fed}{f+d}$, i.e. $Ik \leq Ib$

4 Ergo *k*, falls between *I* and *b*.

CASE II.

l falls between the Eye and *e*.

Draw *go*, *en*, parallel to *fe*, and call $Ie = a$.

1 $If : Ig :: Ie : Io$, i.e. $\frac{fd}{f+d} : d :: a : Io$.

2 Ergo $Io = \frac{af + ad}{f}$,

3 $om = Im - Io = \frac{idf - af - ad}{f}$.

4 $Ik : Ie :: Ic : In$, i.e. $\frac{fed}{f+ed} : ed :: a : In$.

5 Ergo $In = \frac{af + aed}{f}$.

Let $ge = b$, and $gm = ub$, Ergo $em = ub - b$.

6 $gm : om :: em : nm$, i.e. $ub : \frac{idf - af - ad}{f} :: ub - b : nm$

7 Ergo $nm = \frac{idfu - idf - afu + af - adu + ad}{uf}$.

8 $In + nm = id$, ergo multiplying by uf.

9 $iduf = uaf + naed + idfu - idf - afu + af - adu + ad$.

Then expunging and tranſpoſing.

10 $idf = uaed - adu + af + ad.$

11 Ergo $a = \frac{idf}{ued - ud + f + d} = Il.$

Draw mp parallel to Ig, and produce Ie, till it cut mp in p. Then draw pq, parallel to mg, and produce Ig cutting pq in q. Then taking $Ir = Im$, and $Is = Ip$, draw mr and ps.

Then $qp = gm = ub.$

12 $ge : qp :: Ig : Iq :: Ie : Ip.$

13 $b : ub :: d : Iq$, ergo $Iq = ud.$

14 $b : ub :: ed : Ip$, ergo $Ip = ued.$

15 Then becauſe Irm, Isp, are each an Iſoſceles Triangle, and becauſe the Angle $mIr > pIs$, ergo the Angle $Irm < Isp$; but becauſe by reaſon of paralleliſm, the Angles at g and q are equal, and $mg = pq$, ergo the Angle $gmr > qps$, ergo the Subtenſe $gr > qs$.

16 $gr = id - d$, and $qs = ued - ud$, ergo $id - d > ued - ud$,

17 Ergo $id > ued - ud + d$, ergo $id + f > ued - ud + f$

18 Ergo $\frac{idf}{id + f} < \frac{idf}{ued - ud + d + f}$, i.e. $Il < Ie$.

19 Ergo, l falls between the Eye and c.

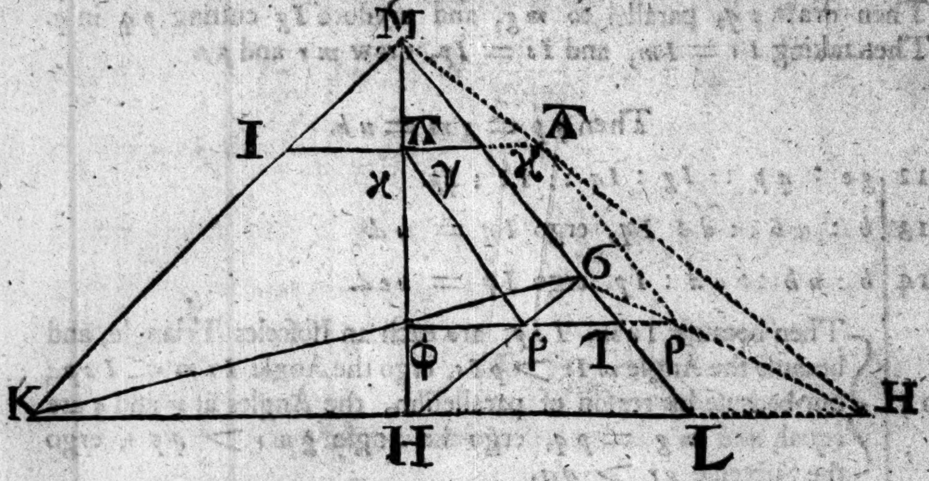
Draw IT parallel to gm, and $\gamma\mu$ parallel to gm alſo; and draw $\phi\kappa\lambda$ correſpondent to fkl.

COROLLARY.

Hence the Line gm is ſeen as a Curve Line, concave toward IT, and convex towards gm, and the farther gm be produced, ſtill the more remote Parts ſeem to approach nearer and nearer to IT; and the like is to be ſaid for $\gamma\mu$.

And this appears in ſome ſort, by two parallel ſtreight Walls, or parallel ſtreight rows of Trees, repreſented by gm, $\gamma\mu$, whoſe more diſtant Parts ſeem to approach nearer and nearer to IT. And ſo the reaſon of the ſeeming approach of the Diſtant Parts of ſuch Lines is given.

Make a Triangle any how KML, and let $I\gamma$, KL, (parallel Lines) cut the Sides of it in I, K, γ, L.



Then taking the Point τ, any where between γ and L, draw $\tau\phi$ parallel and equal to $I\gamma$, and draw $K\phi\sigma$, cutting ML in σ.

Then from any Point of KL, either within or without L, *viz.* from H, draw $H\sigma$, cutting $\phi\tau$ in ϱ; and draw $\varrho\pi$ parallel to ML, cutting $I\gamma$ in π.

Then draw $H\pi$, and produce it as far as M.

Lemma.

$H\pi$ produced will paſs through the interſection of KM and LM.

Draw HM, cutting $I\gamma$ in κ.

1 Then is $KL : LH :: I\gamma : \gamma\kappa$.

2 But $KL : LH :: \phi\tau\ (I\gamma) : \tau\varrho$.

3 Ergo $\tau\varrho = \gamma\kappa$.

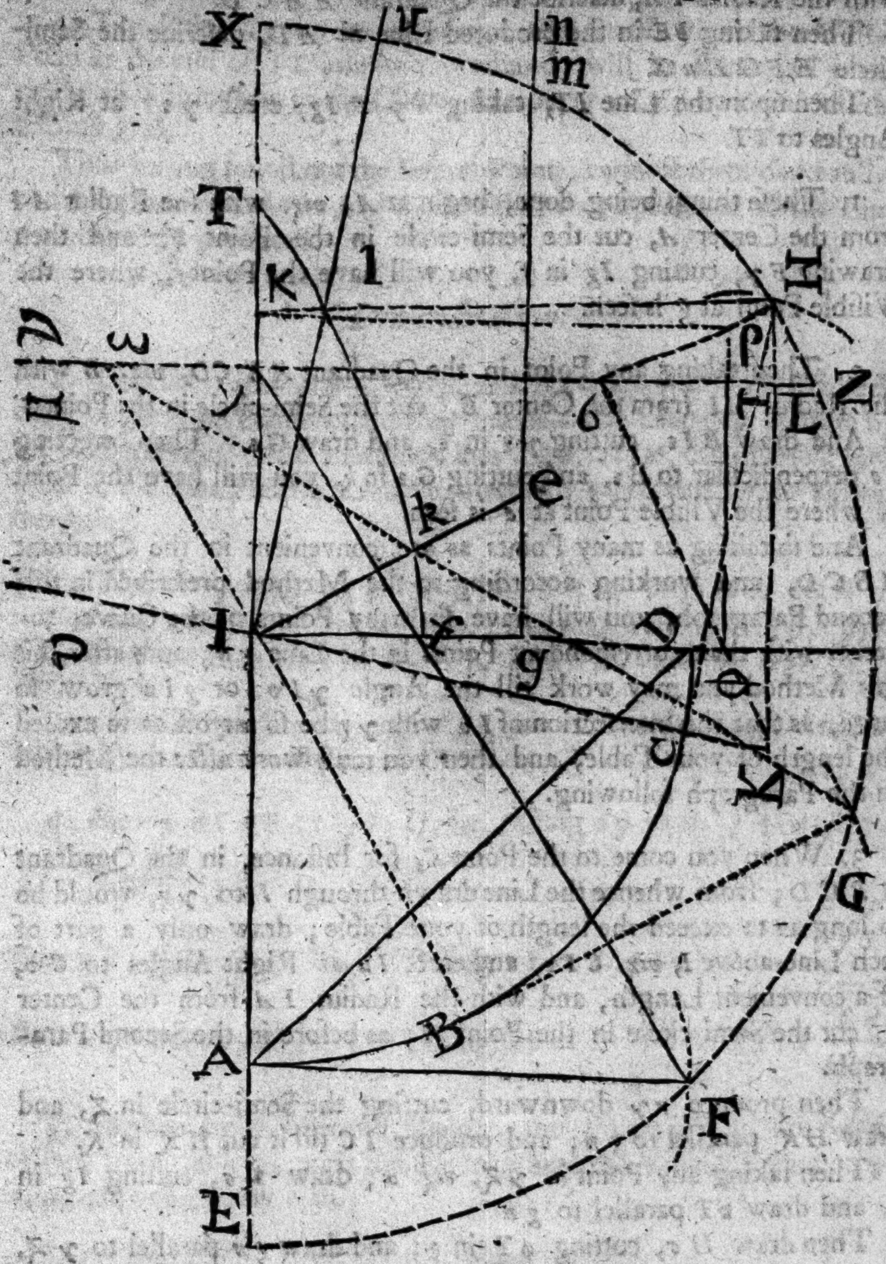
4 But becauſe of the Paralleliſm of $I\gamma$ and $\phi\tau$, and alſo of $\varrho\pi$ and ML, $\varrho\tau = \pi\gamma$.

5 Ergo $\pi\gamma$ and $\gamma\kappa$ is all one, and conſequently $H\pi$ produced, (being the ſame with $H\kappa$ produced) paſſeth through the Interſection of KM with LM.

Probl.

Problem III.

Let the Eye be at *I*, and at the Diſtance of *g*, let $\bar{g}\bar{n}$ of an Infinite



Length, be ſuppoſed to be erected at Right Angles to *Ig*.

Then let it be required to delineate the Curve Line $f k l T$, which ſhall exhibit the appearance of the Line $g n$ to the Eye at I.

Aſſume $IT = f$, and erect IT parallel to $g n$.

Then produce TI. and out of the produced Line taking $IA = IT$, with the Radius IA, deſcribe the Quadrant $A B C D$.

Then taking IE in the produced Line $= AD$, deſcribe the Semi-circle $E F G H u X$.

Then upon the Line IT, taking $I\gamma = Ig$, erect $\gamma \varepsilon \nu$ at Right Angles to IT.

1. Theſe things being done, begin at A, *viz.* with the Radius AI from the Center A, cut the Semi-circle in the Point F, and then drawing $F\gamma$, cutting Ig in f, you will have the Point f, where the Viſible Point at g is ſeen.

2. Then taking any Point in the Quadrant $A B CD$, *viz.* B with the Radius AI from the Center B, cut the Semi-circle in the Point G.

And draw $BI\varepsilon$, cutting $\gamma \nu$ in ε, and draw $G\varepsilon$. Then erecting Ie perpendicular to $B\varepsilon$, and cutting $G\varepsilon$ in k, you will have the Point k, where the Viſible Point at e is ſeen.

And ſo taking as many Points as are convenient in the Quadrant $A B C D$, and working according to the Method preſcribed in this Second Paragraph, you will have ſo many Points of the Curve, together with their correſpondent Points in the Line $g n$; and after this this Method you may work till the Angle $\gamma I v$, or $g I u$ grow ſo large, as that the Interſection of $I v$ with $\gamma \nu$ be ſo far off as to exceed the length of your Table, and then you muſt work after the Method in the Paragraph following.

3. When you come to the Point C, for Inſtance, in the Quadrant $A B C D$; from whence the Line drawn through I to $\gamma \nu$, would be ſo long as to exceed the length of your Table; draw only a part of ſuch Line above I, *viz.* $C I v$, and erect $I u$ at Right Angles to $C v$, of a convenient Length, and with the Radius IA from the Center C, cut the Semi-circle in the Point H; as before in the Second Paragraph.

Then produce $\nu \gamma$ downward, cutting the Semi-circle in Z, and draw HK parallel to $g n$; and produce IC till it cut HK in K.

Then taking any Point in γZ, *viz.* σ; draw $K\sigma$, cutting Ig in φ, and draw φT parallel to $g n$.

Then draw $H\sigma$, cutting φT in ϱ; and draw $\varrho \pi$ parallel to γZ, cutting IT in π.

And then draw $H\varpi$, cutting Iu in l.

Call the Point of Interſection of Iu with gn, m; and call the Point of Interſection of Iv with γv, μ.

Then will l be the Place where the Viſible Point at m is ſeen.

4. The Point T may be taken for the Place where the Viſible Point at the end of IT infinitely produced, will be ſeen; IT being really an Aſſymptote to the Curve, becauſe $g \leq f$; by *Corol.* of the Second *Prob.*

Thus having found out the ſeveral Points, connect them dextrouſly, and you have the Curve $fklT$ exhibiting the appearance of the Line gn, infinitely produced.

Demonſtration.

1. That the Lines $AI\gamma$, $Ff\gamma$, and $BI\varepsilon$, $Gk\varepsilon$, do interſect each other in the ſame Points where they interſect the Line $v\gamma$ appears by the Conſtruction. And that the Lines CIv, $Hl\varpi$, do interſect each other in the ſame Point where they interſect $v\gamma$, appears by the *Lemma* foregoing.

2. That the Angles IAF, or γAF, IBG, or εBG, ICH or μCH are each Right Angles, appears in that IA, AF, IB, BG; IC, CH, are each $= IA$, and IF, IG, IH, each $= AD$.

3. Alſo that γIf, εIk, and μIl, are each equal Right Angles, is plain by the Conſtruction.

4. *Ergo* $\gamma A : AF :: \gamma I : If$, i.e. calling $I\gamma = d$. $f + d : f :: d : \frac{fd}{f+d}$.

Ergo $\varepsilon B : BG :: \varepsilon I : Ik$, i.e. calling $\varepsilon I = d$. $f + d : f :: d : \frac{fd}{f+d}$

Ergo $\mu C : CH :: \mu I : Il$, i.e. calling $\mu I = d$. $f + d : f :: d \; \frac{fd}{f+d}$

5. So then by the Second *Problem*, the] Points f, k, l, are equidiſtant from the Eye, with the Places where the Viſible Points at γ, ε, μ, are reſpectively ſeen.

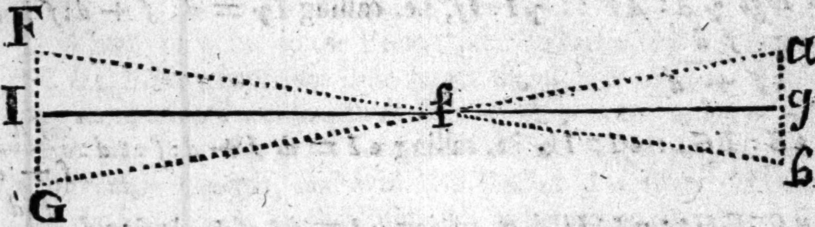
6. But $Ig = I\gamma$, by the Conſtruction and the Angle $Ign = I\gamma\nu$, by the Conſtruction alſo, and becauſe the Angles $eI\varepsilon$, $mI\mu$, are each Right Angles; *ergo* the Angle $eIg = \varepsilon I\gamma$, and the Angle $mIg = \mu I\gamma$; *ergo* $Ie = I\varepsilon$, and $Im = I\mu$; and conſequently the Viſible Points at g, e, m, are reſpectively ſeen in f, k, l.

Ergo a Curve drawn through thoſe Points dextrouſly, will exhibit the Line gn, *&c.*

Meaſure in a plain Field, or upon an even ſmooth Down, a ſtreight Line Ig, of about 120 Yards or upward, and at the Point g, draw a ſtreight Line gn, of a Mile in Length, or upward, and at Right Angles to Ig; placing upon the ſtreight Line gn, at convenient Diſtances upright Stakes, of Colour and Bigneſs enough to make them conſpicuous; and then if you ſtand at I, and look on them, they will appear plainly to be a crooked Line: Alſo if you produce gI, as much on the other ſide of I, and at the extremity thereof, erect a Line ſtuck with Stakes, parallel to gn, the Two Lines will appear Two Curves, whoſe Extremities almoſt meet in the Middle. This is to be proved by ſhewing that the Point f is diſtant from I almoſt $\frac{2}{3}$ of IT, as will appear by working according to the Second *Problem*, taking f there $=2500$ Inches, as is found below at the *Corol.* of *Obſ.* IV.

Problem IV.

The Diſtance of the Point f from the Eye at I, being given to find the Diſtance of g; *viz.* the Place at which the Viſible Point agb, muſt be put to be ſeen in f.



Call $Ig = d$, and call $If = g$, as in *Prob.* II. Then by the 5th Step of *Prob.* II.

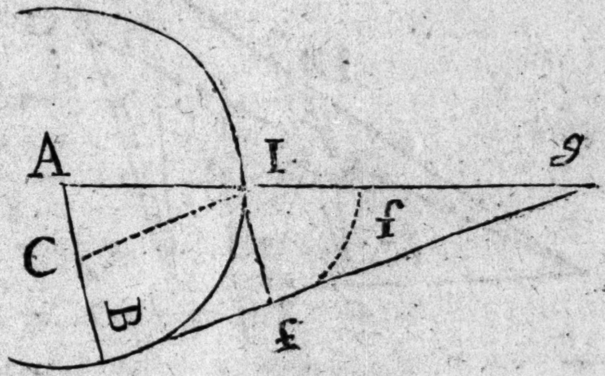
1 | $g = \frac{fd}{f+d}$.

2 | *Ergo* $fd = fg + dg$, and conſequently $fd - gd = fg$.

3 | *Ergo* $d = \frac{fg}{f-g} = Ig$.

Note, That g muſt be leſs than f, by *Corol. Prob.* II.

Geometrical Conſtruction.



Aſſume $AI = f$, and produce it. Then with the Radius AI deſcribe the Circle ABI, and drawing the Radius AB any how, take BC out of it $= If$, and having drawn CI, draw Bg parallel to CI, cutting AI produced in g.

Then will Ig be the Diſtance required.

Demonſtration.

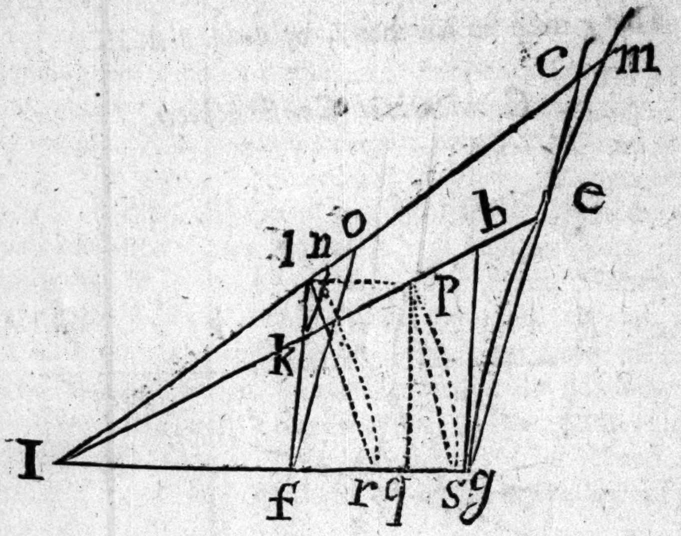
Draw If parallel to AB; *ergo* $If = CB = g$, and $AC = f - g$.

1 | $AC : AI :: If : Ig$, i. e. $f - g : f :: g : \frac{fg}{f-g}$.

2 | *Ergo* $Ig = d$, by the Third Step of *Problem* IV.

Theorem II.

Let the Line meg be expoſed to Sight, the Eye being at I, and let the Viſible Points at m, e, g, be reſpectively ſeen in l, k, f, in the



ſtreight Line fl, the Angle Ifl being not leſs than a Right Angle, and conſequently $Ik > If$, and $Il > Ik$.

Draw gb parallel to fl, and cutting Ie in b, and draw geo, cutting Im in c.

Then will e fall beyond b, and m will fall beyond c.

Demonſtration.

$$\text{Call}\left\{\begin{array}{l} If = g \\ Ik = eg \\ Il = ig \end{array}\right\}\begin{array}{l} \text{Ergo.} \\ e > 1 \\ i > e \end{array}\left\}\begin{array}{c}\text{and then by} \\ \text{Problem II.}\end{array}\right\{\begin{array}{l} Ig = fg \div f - g \\ Ie = feg \div f - eg \\ Im = fig \div f - ig \end{array}\right.$$

CASE I.

e will fall beyond b.

1 | $If : Ig :: Ik : Ib$, i. e. $g : \frac{fg}{f - g} :: eg : Ib$.

2 *Ergo* $Ib = \frac{feg}{f-g}$.

3 But $f-eg < f-g$, *Ergo* $\frac{feg}{f-eg} > \frac{feg}{f-g}$ i. e. $Ie > Ib$.

4 *Ergo* e falls beyond b.

CASE H.

m falls beyond c.

Draw fo, kn, parallel to gc, and call $lc = a$.

1 $Ig : If :: Ic : Io$, i. e. $\frac{fg}{f-g} : g :: a : Io$.

2 *Ergo* $Io = \frac{af-ag}{f}$.

3 $ol = Io - Il = \frac{af-ag-igf}{f}$.

4 $Ie : Ik :: Ic : In$, i. e. $\frac{feg}{f-eg} : eg :: a : In$.

5 *Ergo* $In = af - aeg \div f$.

Let $fk = b$, and $fl = ub$, *Ergo* $kl = ub - b$.

6 $fl : ol :: kl : ln$, i. e. $ub : \frac{af-ag-igf}{f} :: ub-b : ln$

7 *Ergo* $ln = \frac{uaf-af-uag+ag-uigf+igf}{uf}$

8 $In - ln = ig$, *ergo* multiplying by uf.

9 $ufig = uaf - uaeg - uaf + af + uag - ag + uigf - igf$. (See the 5th Step.)

Then expunging and tranſpoſing.

10 $igf = uag - uaeg + af - ag$.

11 *Ergo* $a = \frac{igf}{ug-ueg+f-g} = lc$.

Draw lp parallel to Ig, cutting Ie in p. Then draw pq parallel to fl, cutting Ig in q. Then taking $Ir = Il$, and $Is = Ip$; draw lr, and ps. Then $qp = fl = ub$.

12 $fk : qp :: If : Iq :: Ik : Ip$.

13 $b : ub :: g : Iq$, *ergo* $Iq = ug$.

14 $b : ub :: eg : Ip$, *ergo* $Ip = ueg$.

15 Then becauſe Irl, Isp are each an Iſoſceles Triangle, and becauſe the Angle $lIr > pls$, *ergo* the Angle $Irl < Isp$; but becauſe by reaſon of Paralleliſm, the Angles at f and q are equal, and $fl = qp$; *ergo* the Angle $flr > qps$, *ergo* the Subtenſe $fr > qs$.

16 $fr = ig - g$, and $qs = ueg - ug$, *ergo* $ig - g > ueg - ug$.

17 *Ergo* $ig > ueg - ug + g$, *ergo* $f - ig < f - ueg + ug - g$.

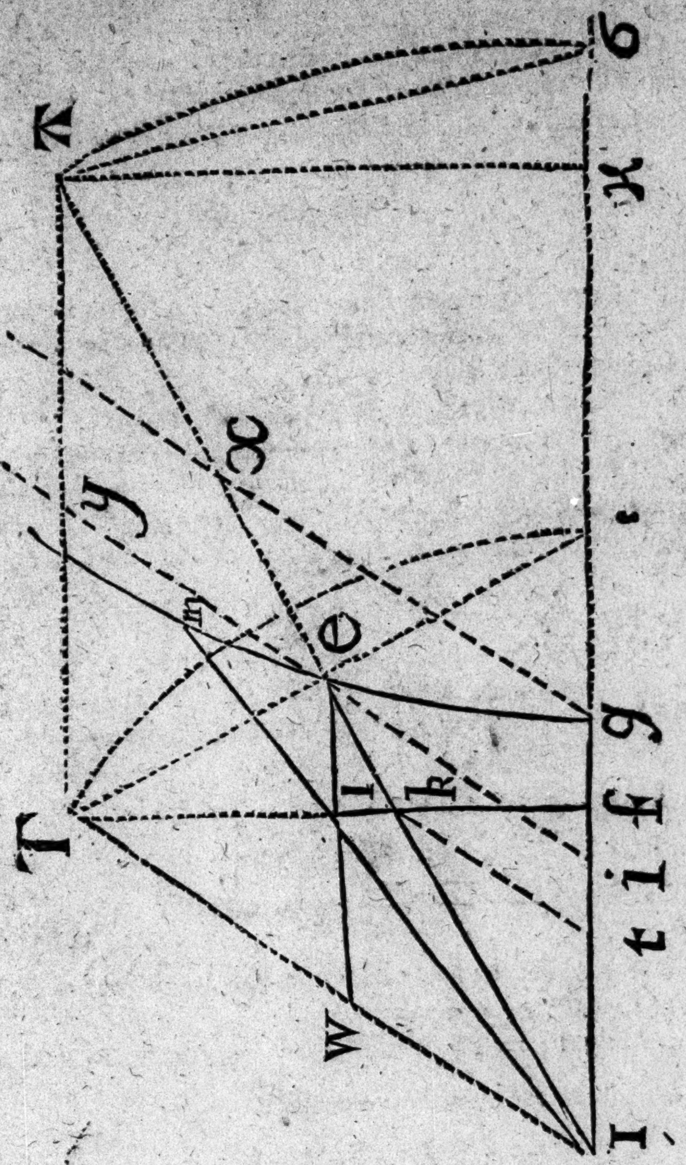
18 *Ergo* $\frac{igf}{f - ig} > \frac{igf}{ug - ueg + f - g}$, i. e. $Im > Ic$.

19 *Ergo* m falls beyond c.

COROLLARY.

Hence the Line gm, which appears as the Streight Line fl, is a Curve Line, convex to fl, and receding more and more from it the Higher it is produced.

Theorem



Theorem III.

Let the Curve *gem*, be expoſed to ſight, the Eye being at *I*, and let the Curve be ſeen as the ſtreight Line *fkT*, ſtanding perpendicularly upon *Ig*.

From the Center I, with the Radius $IT = f$, cut fT in T, and draw IT. Then from g, draw gx, upward and parallel to IT, then will the Curve proceed from g, between the two parallel Lines gx and IT, continually approaching nearer and nearer to IT, which yet it can never come to touch, and ſo IT, may be call'd an Aſſymptote to the Curve.

Demonſtration.

That the Curve can never come to touch IT, though ever ſo much produced, appears by what is gathered from the *Corollary* of the 2d. *Problem*; Namely, that g is always $< f$.

So that it only remains to be proved that the Curve continually approaches nearer and nearer to IT, the higher it is produced.

Let If, Ig, Ik, Ie, ſtand with the ſame Denominations as in the 2d. *Theorem.* Call $fk = b$, and $fT = ub$. *ergo* $Tk = ub - b$.

Let Ike, cut gx in x, and draw kt, parallel to IT, cutting Ig, in t.

1 $Tf : If :: Tk : It$. i. e. $ub : g :: ub - b : It$.

2 *Ergo*, $It = \dfrac{ug - g}{u}$

3 $It : Ik :: Ig : Ix$; i. e. $\dfrac{ug - g}{u} : eg :: \dfrac{fg}{f - g} : Ix$.

4 *Ergo* $Ix = \dfrac{efgu}{fu - f - gu + g}$.

Draw $T\pi$ parallel to Ig, and let it cut Ie in π: Then draw $\pi\varkappa$ parallel to Tf; *ergo* $\pi\varkappa = ub$.

Then making $I\varrho = IT$, and $I\sigma = I\pi$, draw $T\varrho$ and $\pi\sigma$.

5 $kf : \pi\varkappa :: If : I\varkappa :: Ik : I\pi$; i. e.

6 $b : ub :: g : I\varkappa$; *Ergo* $I\varkappa = ug$.

7 $b : ub :: eg : I\pi$; *Ergo* $I\pi = ueg$.

8 Then becauſe $I\varrho T$, $I\sigma\pi$, are each an Iſoſceles Triangle, and the Angle ϱIT, $> \sigma I\pi$; *ergo* the Angle $I\sigma\pi$ $> I\varrho T$. But the Angles at f and $\varkappa$ are equal, as are alſo the Sides fT, $\varkappa\pi$; *Ergo* the Angle $fT\varrho > \varkappa\pi\sigma$; *ergo* the Subtenſe $f\varrho > \varkappa\sigma$.

$f\varrho =$

9 $fg = f - g$, and $ue = ueg - ug$; *Ergo* $f - g > \overline{ueg} - ug$. *Ergo* $f - g + ug > ueg$; *ergo* $f - g + ug - fu > ueg - fu$. *Ergo* $fu - ueg > fu - f - gu + g$.

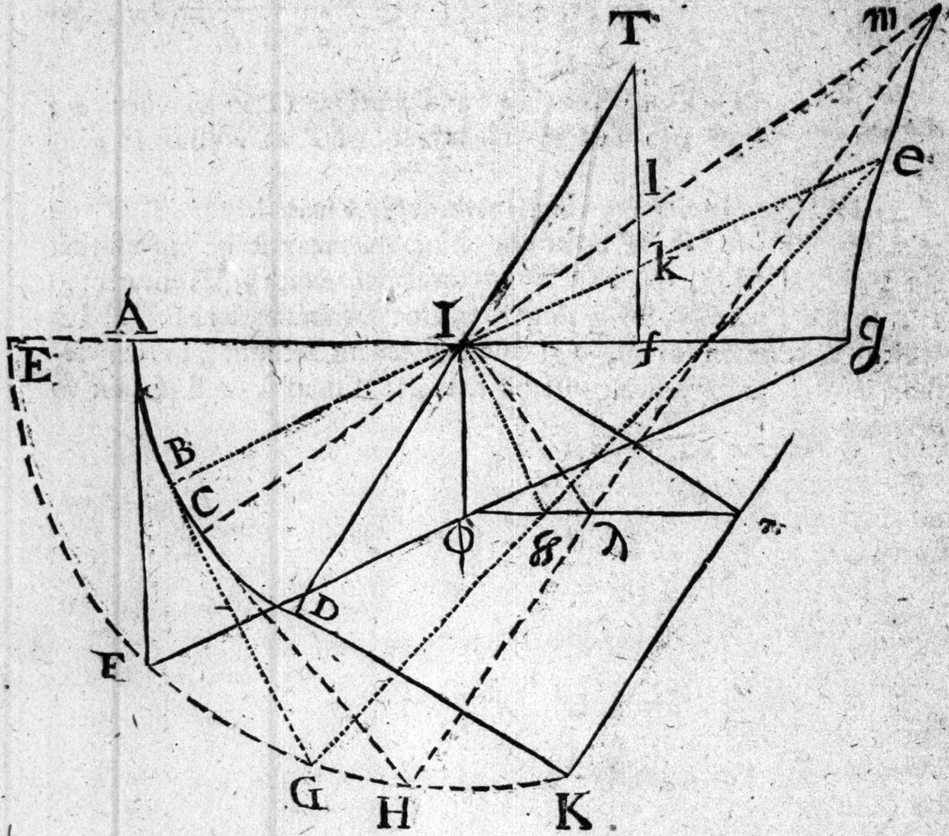
10 *Ergo* $\dfrac{efgu}{fu - ueg} < \dfrac{efgu}{fu - f - gu + g}$; *i. e.* becauſe $\dfrac{efgu}{fu - ueg} = \dfrac{feg}{f - eg} = Ie$, and $\dfrac{efgu}{fu - f - gu + g} = Ix$, *per* 4*th* Step; *ergo* $Ie < Ix$.

11 Draw *ew* parallel to *Ig*, and cutting *IT* in *w*, then $we < Ig$; *ergo* the Curve is nearer to *IT* at *e*, than at *g*.

And in like manner drawing *iy* through *e* parallel to *IT*, if you take the Point *l*, above *k*, in the Line *fT*, may it be proved that the Point *m*, in the Curve which correſponds with *l*, is nearer to *IT* than *e*; namely, by giving the ſame Denominations to *Il*, *Ie*, which were given to *If*, *Ig*, and proceeding according to the Method uſed here for proving *e* to be nearer *IT* than *I*, and ſo on *in infinitum*.

Problem V.

Let the Eye be at *I*, and at the Diſtance of *f*, let *f T* be erected at Right-angles to *I f*.



Then let it be required to delineate the Curve *g e m*, which ſhall appear as the ſtreight Line *f T*, and in the Poſition given.

Aſſume *IT* = *f*. Then with the Radius *IT*, from the Center *I*, cut *fT* in *T*, and draw *T I D*.

Then producing *f I*, from the Center *I*, with the Radius *I T*, draw the Arch *A B C D*, cutting *f I* in *A*, and *T I* in *D*.

Then with the Radius *IE* = the Diameter of the Square of *I T*, from the Center *I*, draw the Arch *E F G H K*, cutting *f I* in *E*.

Erect I φ at Right-angles to I f, and = I f; and at φ erect φ τ at Right angles to I φ, or parallel to I f, as in the Scheme.

1. Theſe things being done, begin at A, *viz.* with the Radius I T, from the Center A, cut the Arch of the Semi-circle in the Point F, and then drawing F φ cutting I f, produced in g, you will have the Point g, *viz.* the Place at which the viſible Point muſt be put to be ſeen in f.

2. Then taking any Point in the Arch A B C D, *viz.* B, with the Radius A I or I T, from the Center B, cut the Arch of the Semi-circle in G.

And draw B I k e, cutting f T in k.

Then erecting I κ perpendicular to B I k, and cutting φ τ in κ; draw G κ e cutting B I k in e: So you will have the Point e, *viz.* the Place at which the viſible Point muſt be put to be ſeen in k.

And ſo taking as many Points as are convenient in the Arch A B C D, and working according to the Method preſcribed in this Second Paragraph, you will have ſo many Points of the Curve anſwering to ſo many in the Line f T.

So finding as many Points, g, e, m, as is convenient, and connecting thoſe Points dextrouſly, you may exhibit as much of the Curve as is needful.

Demonſtration.

All that is to be demonſtrated, is, Firſt, That by the Method preſcribed the Points g, e, m, &c. will be rightly determined; and, Secondly, That it is not needful to continue the Arch A B C D farther than its Interſection with T I D.

For the Firſt then, That the Angles I A F or g A F, I B G or e B G, I C H or m C H are each Right-angles, appears in that I A, A F; I B, B G; I C, C H, are each = I A or I T, and I F, I G, I H, each = the Diameter of the Square of I T, *viz.* = I E.

2. Alſo that the Angles g I φ, e I κ, m I λ, are each Right-angles, appears by the Conſtruction: *Ergo* I φ parallel to A F, I κ parallel to B G, &c.

3. *Ergo* $AF : Ag :: I\varphi : Ig$; i. e. calling $Ig = d$, and $I\varphi = g$. $f : f + d :: g : d$. *Ergo* $fd = fg + dg$. *Ergo* $fg = fd - gd$. *Ergo* $d = \frac{fg}{f - g}$. *Ergo* $I\varphi$ being $= g$, Ig will be $= d$, by *Probl.* 4. And in like manner $I\kappa$ being $= g$, Ie will be $= d$, and $I\lambda = g$, Im will be $= d$.

4. But becauſe, by the Conſtruction, $I\varphi = If$, and the Angles $fI\varphi$, IfT, $I\varphi\tau$, each equal to a Right-angle; and alſo the Angles $kI\kappa$, $lI\lambda$, each equal to a Right-angle: *Ergo* the Angle $kIf = \kappa I\varphi$, and the Angle $lIf = \lambda I\varphi$; *ergo* $Ik = I\kappa$, and $Il = I\lambda$; *ergo* the viſible Points which muſt be ſeen at f, k, l, muſt be placed at g, e, m reſpectively.

Ergo the Curve drawn through the Points g, e, m, as directed, will appear as the ſtreight Line fT, or as a Part of it, and in the Poſition given.

For the Second Thing, That it is not needful to continue the Arch $ABCD$ farther than its Interſection with TID.

At the Point D, putting one Foot of the Compaſſes from thence as from a Center, with the Radius IT cut the Part of the Semicircle in K; erect $I\tau$ at Right-angles to ID, cutting $\varphi\tau$ in τ; and draw $K\tau$ and produce it.

Then becauſe the Angles IDK, $DI\tau$ are Right-angles, *ergo* DK parallel to $I\tau$; and becauſe the Angle $TIf = \tau I\varphi$, *ergo* $I\tau = IT = DK$, *ergo* $K\tau$ parallel to DT. *Ergo* $K\tau$ can never interſect DT, but according to this Method $I\tau$ would be put for g, $= IT = f$, which is contrary to the *Corollary* of the *Second Problem*; and alſo the Point of the Curve anſwering to $I\tau$ or IT, would be in the Interſection of $K\tau$ with DT, which, by reaſon of their Paralleliſm, is impoſſible. *Ergo* all Points of the Curve are determined by Lines drawn from between A and D through I, and conſequently it is not needful to continue the Arch any farther.

Corollary

Corollary.

Hence no Line can appear longer than *f* T at the Diſtance of *f*, and in the Poſition of *f* T with I*f*.

Scholium I.

Let the Line *g e m*, (*Fig.* 1.) infinitely produced, be ſeen as the Curve *f, k, l* T, *&c.* and let *e m*, a part of *g m*, be ſeen as *k l*, part of that Curve; then is the Length of the ſtreight Line *k l*, equal to the Diſtance of the Two Points *k* and *l*, in the Triangle I*k l*.

Fig. 1.

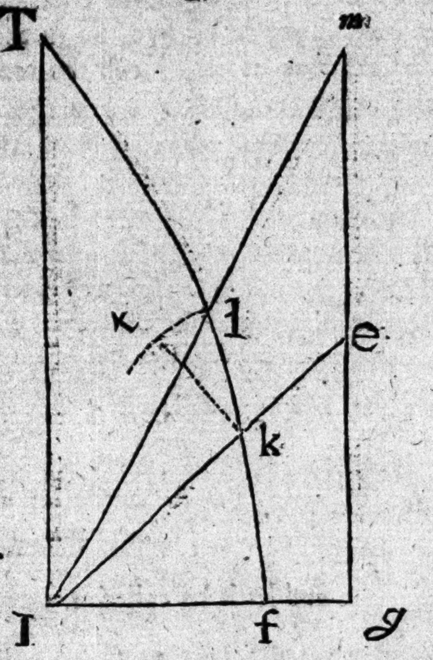
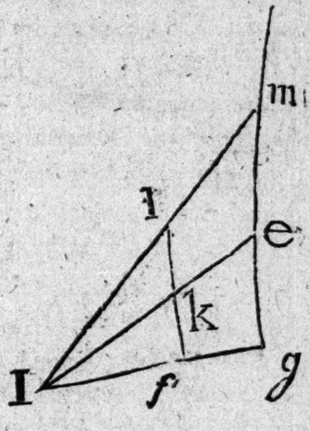


Fig. 2.



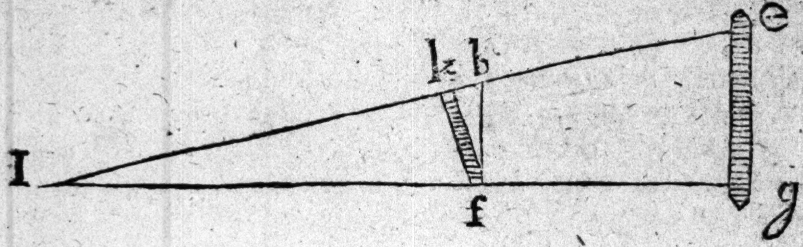
D E F I N I T I O N.

And this Diſtance between the Two Points, I call the local Length of *k l*.

Draw *k t* at Right Angles to I*k*, and equal to *k l*; Then will *e m*, of a few Yards Length, be ſeen more obliquely by the Eye than if *kl* were in the Situation of *kt*, and conſequently as may be gathered by the Firſt Obſervation, *kl* will not appear ſo long as *kt*, that is not ſo many Inches Long. And again, tho' *em* were ſo placed a- to cauſe *kl* to be ſeen in the Situation of *kt*, neither then would it appear ſo many Inches Long as really the Diſtance between the Tw Points *k* and *l* is off, by reaſon of the obliquity of the Parts near *l*; but the Number of Inches which *kl* ſeems to be off, muſt be determined by the Method ſet down in *Problem* I.

DEFINITION.

And this Number of Inches which *kl* ſeems to be of by the Method of the Firſt Problem, I call the meaſured Length of *kl*: So in the Second Figure (all Things being ſuppoſed as before) I call the real Diſtance between *k* and *l*, the local Length of *kl*, &c.



Scholium II.

Let Ig be aſſumed equal to $\frac{f}{3}$ and let it be called d.

1 Then $fd = \frac{ff}{3}$ and $f + d = f\frac{+f}{3} = \frac{4f}{3}$.

2 Then $\frac{f.d}{f+d} = \frac{ff}{4f} = \frac{f}{4} = If$, by Problem II.

Let *ge* of a ſmall Magnitude, ſuppoſe Four Inches in Length be placed perpendicularly upon *Ig* and at *f*, where the Viſible Point at *g* is ſeen; draw *fb* parallel to *ge*, and let the Viſible Point at *e* be ſeen in *k*.

Then the Line *fk* will be a Subtenſe to the Curve *fklT*, (See *Fig.* 1. of *Schol.* 1) whoſe Point *k* in ſtrictneſs falls between *l* and *b*, by *Theorem* I. but in ſo ſhort a Length, the Difference between k and b will be ſo inconſiderable, as that fk may be eſteemed parallel to ge; ſo that as $Ig : If :: ge : fk$.

3 | i. e. $\frac{f}{3} : \frac{f}{4} :: 4 : 3$, *ergo* the local Length of fk, will be equal to Three Inches.

And becauſe kf ſtands perpendicularly upon If, and muſt be at ſome conſiderable Diſtance from I, conſequently the obliquity of its Parts near k will be ſo ſmall, as that the local Length of fk, and the meaſured Length may be eſteemed equal.

OBSERVATION IV.

I drew upon a Board a Column ge, of an Inch and $\frac{1}{3}$ Broad, and Four Inches Long, which Board I ſo placed, as that ge was perpendicular to the horizontal Plane, and of the heighth of my Eye. Then I went off at a Diſtance in a plain Field, and obſerved till the Column ge appeared equal in Length and Bredth to a Column of One Inch broad, and Three Inches long, which I carried in my Hand, holding it as upright as was poſſible, and as near to my Eye as could well be; and marking the place where it ſeemed to be nearly ſo, I meaſured, and found it to be Twenty three Yards Five Inches nearly. *Ergo*

COROLLARY.

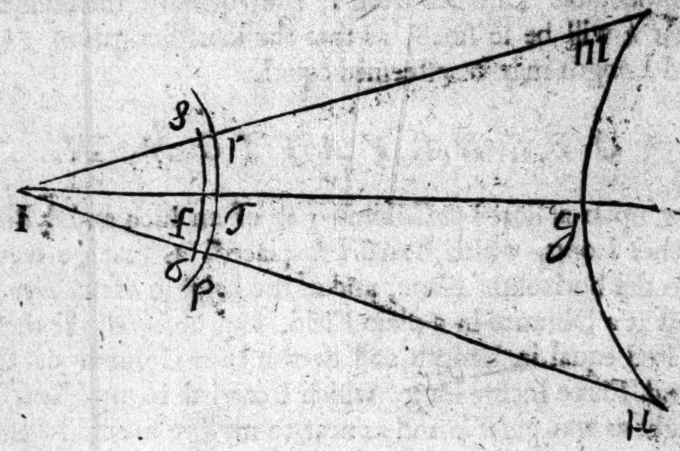
1 | $Ig = 833$ Inches $= \frac{f}{3}$ by the Third Step of *Scholium* II.
2 | *Ergo* $f = 2499$ Inches, or $f = 2500$ Inches nearly.

And thus the f, which hitherto hath been aſſumed, is found, and by that the Calculations and Conſtructions in the former Problems muſt be determined.

1 *Note*, That as I have nearly obſerved the leaſt Magnitude which is capable of being ſeen, or every Viſible Point is ſeen under an Angle of $1''$, and ſo half the Bigneſs of the Viſible Points at g and e, which are ſeen in f and k reſpectively, are below and above Ig, Ie, reſpectively, namely the Part of the Viſible Point below Ig, muſt be ſo Big as to be the Subtenſe of $30'''$ at the Diſtance of Ig, &c. and muſt be added to eg, &c. accordingly.

And this Note muſt be underſtood to be applicable to all like Caſes following in this Treatiſe.

2 *Note*, After this Method f, may be found out many ways, namely, by putting $Ig = \frac{f}{2}$ &c. but I choſe this way, becauſe the Eye cannot ſo nicely diſtinguiſh at a greater Diſtance.



Scholium III.

Let $If = g$, and $Ig = d$, and let $If = f$ — One Yard; *i. e.* let $If = 2464$ Inches.

Then $\frac{fg}{f-g} = \frac{6160000}{36} = 171111$ Inches $= 4753 +$ Yards $= Ig$. *Per Prob.* IV.

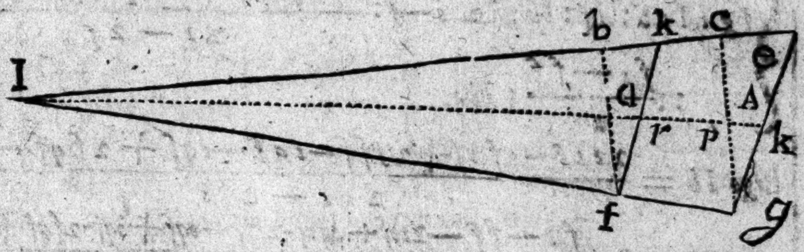
Draw gm, $g\mu$, Curvilinear Lines convex toward I, and taking $IT = f$, with the Radius IT; from the Center I, draw the Arches Pr, $T\varrho$, and draw Im, $I\mu$. Then if gm, $g\mu$ are produced ever ſo long, all their Parts will be ſeen within the Arches Tr, $T\varrho$, becauſe the Viſible Points at m and μ can never be ſeen ſo far off from I, as r and ϱ, by *Corol. Prob.* II. and conſequently the Curve $mg\mu$, will appear to the Eye circular, like the Arch sfv.

COROLLARY.

Hence it is, that whatever Parts of the Earth are ſeen at Three Miles Diſtance, or thereabout, they appear circular, be they extended upon a ſtreight Line, or running out into Angles, or Curve any manner of way; and this is agreeable to Experience, becauſe whereever you are; the Hills or Plains which bound your Sight, provided they be at the Diſtance ſpecify'd, or beyond it, do appear to be round and like a Circle.

Problem VI.

The Diſtances of the Points *g* and *e*, from the Eye at *I* being given, together with the Length of the Line *g e*, (the Angle *I g e*, being not leſs than a Right Angle) to find the local Length of the Line *f k*, connecting the Points *f* and *k*, where the Viſible Points at *g* and *e* are reſpectively ſeen. (*See the Note following* Prob. VII.)



Call $\left\{\begin{matrix} Ig = d = s - f \\ Ie = ed = t - f \end{matrix}\right\}$ *ergo* $\left\{\begin{matrix} s = f + d \\ t = f + ed \end{matrix}\right\}$ *ergo* by *Prob.*2. $\left\{\begin{matrix} If = \frac{fd}{f+d} = \frac{fs - ff}{s} \\ Ik = \frac{fed}{f+ed} = \frac{ft - ff}{t} \end{matrix}\right.$

Draw *g c* and *f b* perpendicular to *I e*.

Call $\left\{\begin{matrix} ge = k \\ fk = r \end{matrix}\right\} \left\{\begin{matrix} gc = p \\ ce = a \end{matrix}\right\}$ *ergo* $Ic = t - f - a$.

1 | $kk = pp + aa$, and $\Box\, Ig = pp + \Box\, Ic$. *Ergo* $\Box\, Ig - kk = \Box\, Ic - aa$.

2 | i. e. $ss - 2sf + ff - kk = tt - 2tf - 2ta + ff + 2fa$. *Ergo*

3 $2ta - 2fa = tt - 2tf - ss + 2sf + kk$. *Ergo*

4 $a = \frac{tt - 2tf - ss + 2sf + kk}{2t - 2f}$.

Let $t - s = q$; i. e. becauſe $t - s = ed - d$. $q = ed - d$.

Then $tt - ss = tq + sq$. And $-2tf + 2sf = -2fq$.

An then changing the Denominations accordingly.

5 $a = \frac{tq + sq - 2fq + kk}{2t - 2f}$.

6 $Ic = t - f - a$; i. e. bringing to the Denom. of a, $Ic = \frac{2tt - 4ft + 2ff - tq - sq + 2fq - kk}{2t - 2f}$;

7 $Ig : Io :: If : Ib$; i.e. $s - f : \frac{2tt - 4ft + 2ff - tq - sq + 2fq - kk}{2t - 2f}$ $:: \frac{fs - ff}{s} : Ib$.

8 *Ergo* $Ib = \frac{2ttf - 4ftf + 2fff - tqf - sqf + 2fqf - kkf}{2ts - 2fs}$

9 $bk = Ik - Ib = \frac{fs - ff}{t} - \frac{2ttf + 4ftf - 2fff + tqf + sqf - 2fqf + kkf}{2ts - 2fs}$

That is, reducing to one Denomination, reducing $t - s$ to q, and expunging.

10 $bk = \frac{-ttfq + 2fftq - 2fffq + sftq + kkft}{2tts - 2tfs}$.

11 $Ig : go :: If : fb$; i. e. $s - f : p :: s - f \times \frac{f}{s} : fb$.

12 *Ergo* $fb = \frac{pf}{s}$; *Ergo* $\square fb = \frac{ppff}{ss}$, and becauſe $pp = kk - aa$.

13 $\square fb = \frac{kkff - aaff}{ss}$ i. e. multi. by $tt = \frac{kkfftt - aafftt}{sstt}$.

Then ſquaring a, according to its Denom. in the 5th Step, &c.

14 $$\left\{\square fb = \frac{kkfftt}{sstt}\left\{\frac{-fft^4qq - 2fft^3sqq + 4fft^3fqq - 2fft^3kkq - ffttssqq + 4ffttsfqq - 2ffttskkq - 4f^4ttqq + 4ffttfkkq - ffttk^4}{4t^4ss - 8t^3fss + 4ffttss}\right.\right.$$

Then ſquaring the 10th Step.

15 $$\left\{\square\, bk = \left\{\frac{t^4ffqq - 4t^3f^3qq + 4f^4ttqq - 2t^3ffsqq - 2t^3ffqkk + 4f^4ttqq - 8f^5tqq + 4f^3ttqqs + 4f^3ttqkk + 4f^6qq - 4f^4qqst - 4f^4qkkt + ssffttqq + 2sffttqkk + k^4fftt.}{4t^4ss - 8t^3fss + 4ttffss}\right.\right.$$

Then expunging and adding together the Squares.

16 $$\left\{\begin{matrix}\square fb \\ + \square bk\end{matrix}\right\} = \frac{kkfftt}{sstt} - \left\{\frac{-4fft^3sqq - 4fft^3kkq + 8f^3ttsqq + 8f^3ttkkq + 4f^4ttqq - 8f^5tqq + 4f^6qq - 4f^4qqst - 4f^4qkkt.}{4t^4ss - 8t^3fss + 4ttffss}\right.$$

Then Dividing by $4tt - 8tf + 4ff$.

17 $$\left\{\begin{matrix}\square fb \\ + \square bk\end{matrix}\right\} = \frac{kkfftt}{sstt} - \frac{fftsqq + f^4qq - fftkkq.}{sstt}$$

But becauſe $t - s = q$, *ergo* $t - q = s$, *ergo* changing accordingly.

18 $$\left\{\begin{matrix}\square fb \\ + \square bk\end{matrix}\right\} = \frac{f^4qq - ffqqts + ffkkts}{sstt} = rr.$$

19 $$\textit{Ergo } r = \frac{f}{st} \times \sqrt{:ffqq + kkts - qqts} \left\{\begin{matrix} s = f + d \\ t = f + ed \\ q = ed - d \end{matrix}\right\} \text{All given Quantities}$$

So reſolving, this Equation r will be found.

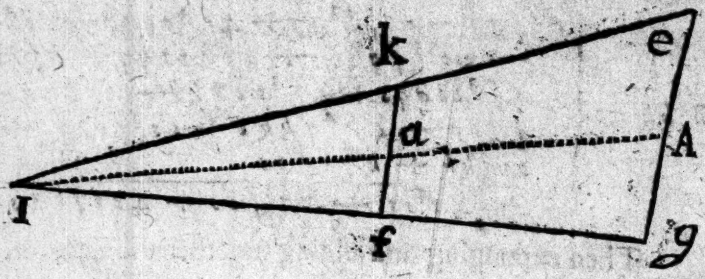
As for the Geometrical Conſtruction :

Having delineated the Given Triangle lge, find out the Points f and k, by the Second Problem, and draw fk.

The

Problem VII.

The Diſtances of the Points f and k, from the Eye at I, being given, together with the local Length of the Line fk, connecting thoſe Points (the Angle Ifk, being not leſs than a Right Angle) to



find the Length of the Line ge, connecting the Points g and e, at which Places Viſible Points muſt be put to be ſeen in f and k, reſpectively. (*See the Note following this Problem.*)

$$\left.\begin{array}{l}\text{Call } If = g = f - x;\ \textit{ergo}\ x = f - g \\ \text{Call } Ik = eg = f - y;\ \textit{ergo}\ y = f - eg\end{array}\right\}\ \textit{Ergo p. Prob. 4}\ \left\{\begin{array}{l} Ig = \frac{fg}{f-g} = \frac{ff-fx}{x}. \\ Ie = \frac{feg}{f-eg} = \frac{ff-fy}{y};\end{array}\right.$$

Call $ge = k$: Call $fk = r$.

Then accomodating to the Denominations in the *Sixth Problem*,

1 $Ig = \frac{ff - fx}{x} = s - f$; *ergo* $s = \frac{ff}{x}$.

2 $Ie = \frac{ff - fy}{y} = t - f$; *ergo* $t = \frac{ff}{y}$.

3 *Ergo* $t - s = q = \frac{ffx - ffy}{xy}$, and $qq = \frac{f^4xx - 2f^4xy + f^4yy}{xxyy}$.

4 And $st = \frac{f^4}{xy}$, and $sstt = \frac{f^8}{xxyy}$.

Then, by the *Eighteenth Step* of the *Sixth Problem*,

5 $rr = \frac{f^4qq - ffqqst + ffkkst}{sstt}$. *Ergo* tranſpoſing and mult.

$ffkkst$

6 $ffkkst = ssttrr - f^4qq + ffqqst.$

That is, changing into the New Denominations

7 $$\frac{f^6kk}{xy} = \frac{rrf^8 - f^8xx + 2f^8xy - f^8yy}{xxyy} + \frac{f^{10}xx - 2f^{10}xy + f^{10}yy}{x^3y^3}.$$

Then multiplying by xy, dividing by f^6, and reducing to one Denomination

8 $$kk = \frac{rrffxy - ffxxxy + 2ffxxyy - ffyyxy + f^4xx - 2f^4xy + f^4yy}{xxyy}$$

Let $x - y = eg - g = z$, and then $xx - 2xy + yy = zz$.

And then changing accordingly

9 $$kk = \frac{rrffxy - ffxyzz + f^4zz}{xxyy}.$$

10 $$\text{Ergo } k = \frac{f}{xy} \times \sqrt{:rrxy - zzxy + ffzz} \quad \left\{ \begin{array}{l} x = f - g \\ y = f - eg \\ z = eg - g \end{array} \right\} \text{All given Quantities.}$$

So reſolving this Equation k will be found.

The Geometrical Conſtruction is had by a bare Delineation of the Triangle Ifk, &c. as in the *Sixth Problem*, *mutatis mutandis*.

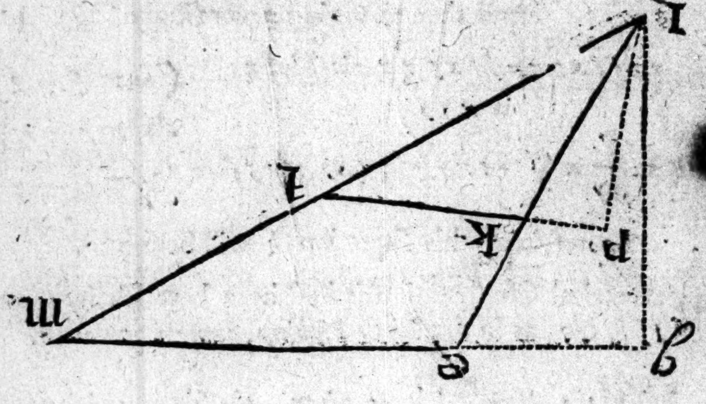
Note, *When the Angle* f I k *is given*, *the* Sixth *and* Seventh Problems *may be performed eaſier*, *by the* Third *and* Second Axioms *of* Plain Trigonometry, *having firſt found out* I g, I e, *or* I f, I k; *by the* Fourth *or* Second Problem, *as the Caſe requires*.

Scholium.

Draw the Line *IaA* cutting *fk* in *a*, and *ge* in *A*, in the Figures of the *Sixth* and *Seventh Problems*. (*See the Figures of the* Sixth *and* Seventh Problems.)

fk, in the Figure of the *Sixth Problem*, is the Subtenſe of a Curve, *viz.* Part of the Curve *fklT* in the Firſt Figure of the *Firſt Scholium* of the *Fifth Problem*. (*See Fig.* 1. of *Schol.* 1. *Probl.* 5.) And in ſtrictneſs, the viſible Point at *A*, in the Line *ge*, is ſeen more diſtant from *I* than *a*. But here the Difference between the Subtenſe *fk* and the Curve, is ſo inconſiderable when *fk* exceeds not Two or Three Yards, and when *If* exceeds not much $\frac{f}{2}$, that it may be taken for the Curve it ſelf: And ſo I ſhall make the Account in like Caſe in the following Problems, taking *a* for the Place where the viſible Point at *A* is ſeen. Alſo I ſhall conſider *fa* as repreſenting *gA*, and *ak* as repreſenting *Ae* making Allowances according to the Firſt Note of Obſervation IV.

For like Reaſons I ſhall take *gA* in the Figure of the *Seventh Problem* to be put to be ſeen as *fa*, and *Ae* to be put to be ſeen as *ak* in like Caſe.



Problem VIII.

Let the Eye be at *I*, and let the Line *em* be ſeen as the Line *kl*, the Angle *Iem* being not leſs than a Right-angle; and let the Length of the Line *em*, and the local Length of *kl* be given: And alſo let either the Length of *Ie* and *Im* be given, or the Length of *Ik* and *Il*. Produce *me*, and let fall *Ig* perpendicular to *me* produced.

Then let it be required to find the Angles *Ikl*, *Iem*, and alſo the Lines *eg*, *Ig* by Calculation.

Call $\left\{\begin{array}{l} Ik = g \\ Il = G \\ kl = r \end{array}\right\}$ Call $\left\{\begin{array}{l} Ie = d \\ Im = D \\ em = k \end{array}\right\}$ Call $\left\{\begin{array}{l} eg = e \\ Ig = P \end{array}\right\}$ Call the Radius $= R$.

Caſe

Caſe I.

Let *Ie*, *Im* be given.

Then, Firſt, for finding the Angle *Ikl*. By the *Second Problem* find ſeverally *Ik*, *Il*; ſo you will have *g* and G.

Then reſolve this Equation in Numbers, *viz.* $\frac{R}{2g.r} \times GG - gg - rr$.

This will be the Co-ſine of the Angle *Ikl*; which Angle being found in the Triangular Canon, the Complement *Ikl* will be found alſo.

Then, Secondly, for finding the Angle *Iem*.

Reſolve this Equation in Numbers, *viz.* $\frac{R}{2kd} \times DD - dd - kk$.

This will be the Co-ſine of the Angle *Iem*; which Angle being found in the Triangular Canon, the Complement *Iem* will be found alſo.

Note, *The Angles* I k l, I e m, *are not leſs than Right-angles, and ſo inſtead of the Angles found for them you muſt take their Supplements, except that* I k l *may be leſs than a Right-angle, when the Angle* e I g *is ſmall.*

Then, Thirdly, for finding *eg*, *Ig*.

$eg = \frac{DD - dd - kk}{2k} = e$, and $Ig = \sqrt{:dd - ee} = P$, *per Prop.*47.*Elem.* 1.

Caſe II.

Let *Ik*, *Il* be given.

Then, Firſt, for finding the Angle *Ikl*. Reſolve the Equation for this in the Firſt Caſe, and proceed by the Method there.

Then, Secondly, for finding the Angle *Iem*.

By the Fourth Problem find ſeverally *Ie*, *Im*; ſo you will have *d* and *D*. Then reſolve the Equation for this in the Firſt Caſe, and proceed as there.

Demonſtration.

Firſt, That the Co-ſine of the Angle $Ikl = \frac{R}{2gr} \times GG - gg - rr$.

Produce lk, and let fall IP perpendicular to lk produced.
Call $kP = y$, and call $IP = p$.

1 $\left\{ \begin{array}{l} GG = pp + yy + rr + 2yr, \text{ and } gg = pp + yy; \textit{ Ergo} \\ GG - gg = rr + 2yr. \end{array} \right.$

2 *Ergo* $y = GG - gg - rr \div 2r$.

3 $g : R :: y :$ Sine $kIP :: GG - gg - rr \div 2r :$ Sine kIP.

4 $\left\{ \begin{array}{l} \textit{Ergo} \text{ the Sine of } kIP = \text{the Co-ſine of } Ikl = \frac{R}{2gr} \times GG \\ - gg - rr. \end{array} \right.$

After the ſame Manner may it be ſhewn, that the Co-ſine of $Iem = \frac{R}{2kd} \times DD - dd - kk$; and alſo that $eg = \frac{DD - dd - kk}{2k}$.

Note 1. *When the Angle* Ikl *is leſs than a Right-angle, then the Co-ſine of* $Ikl = \frac{R}{2gr} \times rr + gg - GG$.

Note 2. *When the Angle* kIl *is given, the Angles* Ikl, Iem *are found out moſt eaſily by the* Third Axiom *of* Plain Trigonometry; *and then* eg *and* Ig *are found out moſt eaſily by the* First Axiom *of* Plain Trigonometry.

Fig. 1.

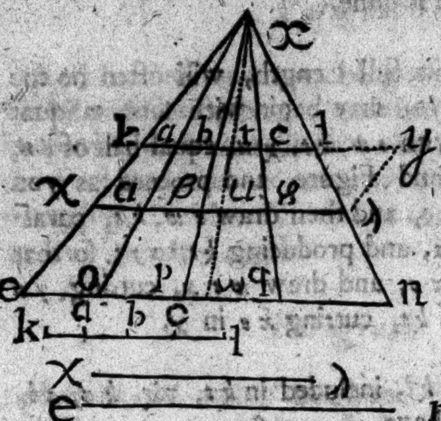


Fig. 2.

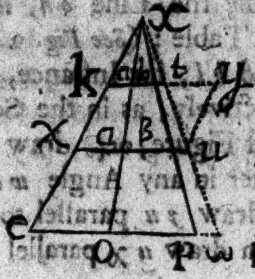
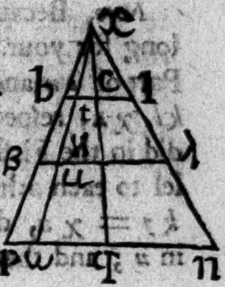


Fig. 3.



Lemma.

Let the Three Lines kl, $\chi\lambda$, en be given in Length; and let kl be divided any how into ka, ab, bc, cl.

Then let it be required to divide $\chi\lambda$ into $\chi\alpha$, $\alpha\beta$, $\beta\kappa$, $\kappa\lambda$, and alſo let it be required to divide en into eo, op, pq, qn, ſo as that $ka : ab :: \chi\alpha : \alpha\beta :: eo : op$. And $ab : bc :: \alpha\beta : \beta\kappa :: op : pq$. And $bc : cl :: \beta\kappa : \kappa\lambda :: pq : qn$.

Draw xke a ſtreight Line in the Firſt Figure, (*See* Fig. 1.) and draw kl making an Angle any how with xe, and produce kl to y ſo as that $ky = \chi\lambda$. Then taking any Point in the Line xe below k, draw en parallel to kl, and draw nl cutting xe in x.

Then draw $y\lambda$ parallel to xe, cutting xn in λ; and draw $\lambda\chi$ parallel to lk, cutting xe in χ.

Then marking the Diviſions given in kl, draw $xa\alpha o$, $xb\beta p$, $xc\kappa q$.

Then will $\chi\alpha$, $\alpha\beta$, $\beta\kappa$, $\kappa\lambda$, and eo, op, pq, qn, be the Diviſions required.

Demonſtration.

That lk, ne, in the Figure, are equal to the given lk and ne reſpectively, appears by the Conſtruction; and $\lambda\chi$, in the Figure, is equal to the given one, becauſe $k\lambda$ is a Parallelogram, and the Side yk oppoſite to $\chi\lambda =$ to the given one.

Then by the 4th Prop. of the 6th Element, *ka* : *ab* :: *χα* : *αβ* :: *eo* : *op*, &c. *ergo* the Thing required is done.

Note, Becauſe the Line *en*, in its full Length, will often be too long for your Table; (*See Fig.* 2.) you may begin with ſome aliquot Part of *en* and *kl*; for Inſtance, with *ew*, *kt*, *χu*, equal half of *en*, *kl*, *χλ*, reſpectively, as in the Second Figure, and proceed as you did in the Firſt Figure, *viz.* draw *xe*, and then draw *ew*, *kt*, parallel to each other in any Angle *wex*, and producing *kt* to *y*; ſo that *ky* = *χu*, draw *yu* parallel to *xe*, and draw *wtx*, cutting *yu* in *u*; and then draw *uχ* parallel to *kt*, cutting *xe* in *χ*.

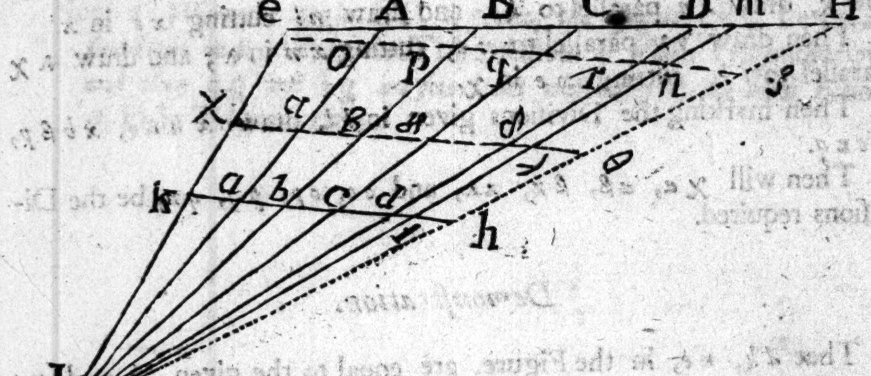
Then Noting thoſe Diviſions of *kl*, included in *kt*, *viz.* *ka*, *ab*, draw *xaαo*, *xbβp*; ſo you will have *χα*, *αβ*, *eo*, *op*, as before.

And then begin again with *xp*, as in the Third Figure, (*See Fig.* 3.) repeating the ſame Proceſs.

The Demonſtration for this is the ſame as before,

Problem IX.

Let the Eye be at *I*, and let the Line *em*, be ſeen as the Line *kl*, the Angle *Iem* being not leſs than a Right Angle, and let either *Ie*, *Im*, and *em*, be given; or *Ik*, *Il*, and *lk* be given.



Then let it be required to divide *em* into *eA*, *AB*, *BC*, &c. ſo as that each of thoſe Diviſions ſhall appear equal in Length to One Inch.

Paragraph I.

Paragraph 1. *Caſe* I.

Let Ie, Im, and em, be given: Find out the local Length of kl by the 6th Problem; and then find the Angle Ikl, by the 8th Problem: And next divide kl by the Segmental, after the Method of the Firſt Problem, or rather after the Method of the Second Scholium foregoing, Problem X. So as that ka, ab, bc, &c. appear each = to One Inch in Length.

Paragraph 1. *Caſe* II.

Let Ik, Il, and kl be given: Find out the Length of em by the 7th Problem; and then find the Angle Ikl by the 8th Problem; and next divide kl by the Segmental, after the Method of Prob. I. or rather by the Method of the Second Scholium of the Second Lemma foregoing, Problem X. So as that ka, ab, bc, *&c.* appear each equal to One Inch in Length.

And then find the Angle Iem by the 8th Problem, (when the Angle kIl is given, find Ie, Im, by the 4th Problem; and then find the Angle Iem, by the Third Axiom of Plane Trigonometry; and then by the Second Axiom of Plane Trig. find em. (*See the Note following the Seventh Problem*).

Paragraph 2.

Theſe Things being done, delineate the Figure $Ikeml$; (which is eaſy to do the Angles Ikl, Iem being found by the 8th Problem; and in order thereunto, the Lines Ik, Ie, being found or given, and the Lines kl, em, given alſo).

And then draw IaA, IbB, IcC, IdD, cutting em, in A, B, C, D. And ſo you have eA, AB, BC, CD, which will each be ſeen = to One Inch in Length, to the Eye at I, and Dm will appear equal to a part of an Inch.

Demonſtration.

That eA, AB, BC, *&c.* are ſeen as ka, ab, bc, *&c.* appears by the Scholium of the 7th Problem, and that ka, ab, bc, *&c.* appear but equal to One Inch each, is plain from Prob. I. conſequently eA, AB, BC, *&c.* appear each equal to One Inch.

Scholium I.

Scholium I.

Produce *k l* to *b*, ſo as that *d b* may appear equal to One Inch, after the Method of Prob. I. and draw *I b H*, cutting *e m* produced in *H*.

In ſtrictneſs, the Line which would be ſeen as the Line *e H*, muſt fall within *k b*, toward *I*, as may be gathered by conſidering the Curve *f k l T* in the Problem; but here the Difference would be ſo ſmall between *k b* and that other Line, that *k b* may be taken for it; and ſo *D H* may be put to repreſent an Inch in Length to the Eye at *I*.

And by a like reaſoning *k d* may be taken, as repreſented to the Eye by *e D*, and part for part, as before.

Scholium II.

Becauſe the Lines *I e*, *I m*, are often very long in Proportion to *k a*, *a b*, *b c*, &c. So that although by the help of a Scale, the Lines *I e*, *I m* might be brought to comply with your Table, yet by reaſon of the ſmallneſs of the Parts *k a*, *a b*, *b c*, &c. the Proportion between them would be very uncertainly drawn; therefore it will be very convenient to proceed after the following Method for the Second Paragraph.

Aſſume ſome conſiderable part of *I e*, *viz.* *I χ*, ſo as that *χ e* may be left as long as will well comply with your Table, conſidered with reſpect to the Angles *e k l*, *k e m*.

Call $I\chi = d$. Call $Ik = g$. Call $kl = r$.

Then reſolve this Equation in Numbers, *viz.* $\frac{dr}{g}$. So you will have $\chi\lambda$, *viz.* a Line cutting the Triangle *e I m* in χ and λ, and parallel to *k l*; becauſe ſince in this Caſe $Ik : kl :: I\chi : \chi\lambda$; i. e. $g : r :: d : \chi\lambda$; *ergo* $\chi\lambda = \frac{rd}{g}$.

Then calling $Ie = D$, reſolve this Equation in Numbers, *viz.* $\frac{Dr}{g}$, ſo you will have *e n*, *viz.* a Line cutting the Triangle *e I m* in *e* and *n*, and parallel to *k l*; becauſe ſince in this Caſe $Ik : kl :: Ie : en$; i. e. $g : r :: D : en$, *ergo* $en = \frac{Dr}{g}$.

Then by the Method of the *Lemma*, divide $\chi\lambda$, en; ſo as that $ka : ab :: \chi\alpha : \alpha\beta :: eo : op$, &c.

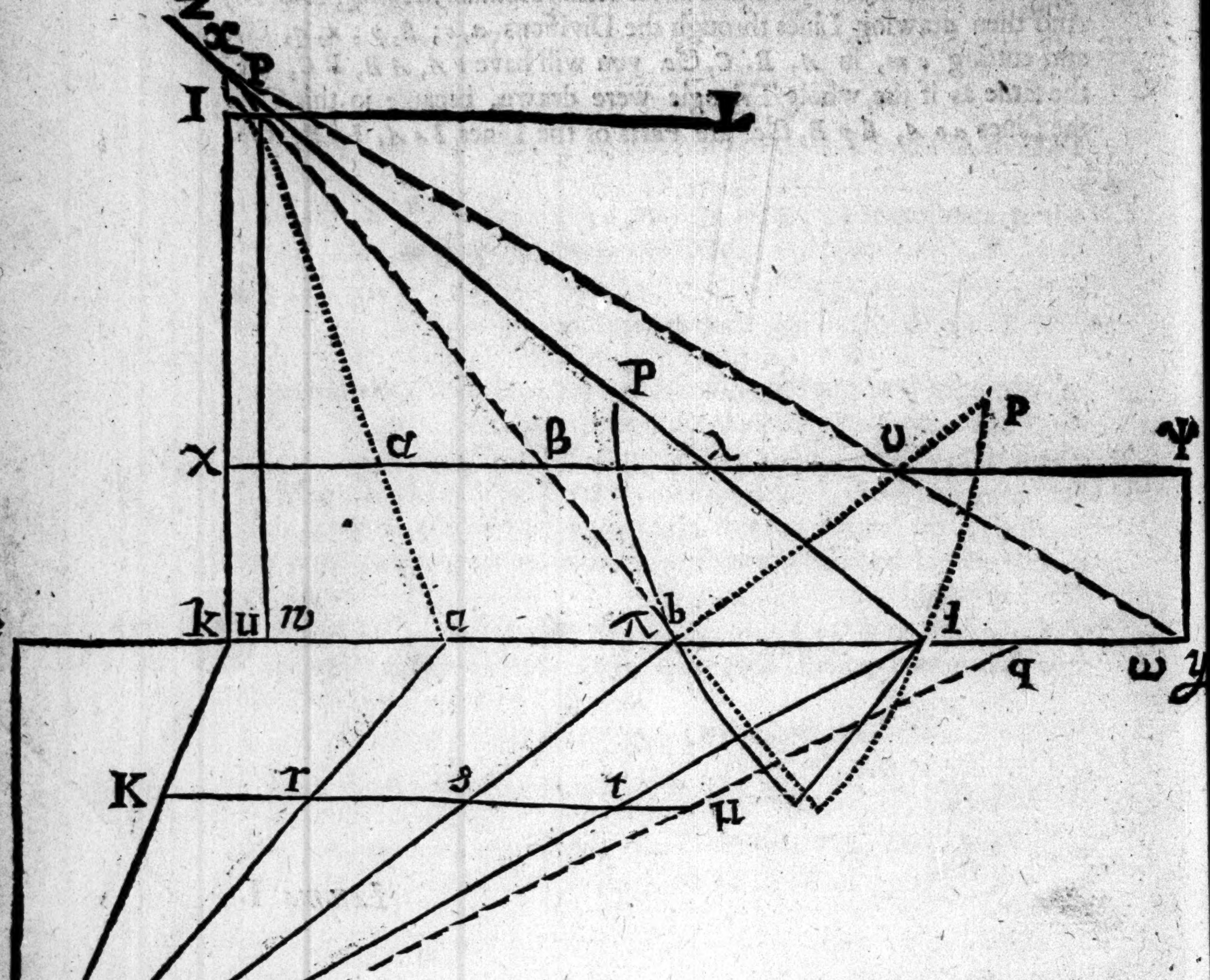
Then drawing $\chi\lambda$, draw χe, ſo as that the Angle $e\chi\lambda = ekl$; and then at e, draw en parallel to $\chi\lambda$.

And at e draw em, ſo as that the Angle χem equal to Iem; found by *Problem* VIII. (*See the Method in the Second Scholium foregoing, Prob.* X.) And then drawing Lines through the Diviſions α, o; β, p; $\varkappa, q$, &c. and cutting em, in A, B, C, &c. you will have eA, AB, BC, &c. the ſame as if the whole Triangle were drawn, becauſe in this Caſe the Lines αoA, βpB, &c. are Parts of the Lines IaA, IbB, &c.

Lemma I.

Lemma I. *Theorem.*

Let the Eye be at I, and let the Line *f l* be divided according
he Method of *Prob.* I. the Angle *I f l* being a Right Angle, an



let *f k*, *k a*, *a b*, *b l*, each be ſeen of an Inch in Length:

Er

Erect at k, kx, perpendicular to fl; and taking $kJ = If$, draw JL parallel to fl.

Take $lw = fb$, and erect wp perpendicular to fl, and cutting JL in p, and draw lp.

Then apply the Segmental to lp, *viz.* the Rectangular Point at l, and the Longeſt Side toward p, ſo as that the Curve Side of it may cut fl in ϖ, *viz.* toward f.

Then is $l\varpi = bl$.

Demonſtration.

Becauſe $lw = bf$, and the Angle $lwp = bfI$, and the Side $wp = fI$; *ergo* the Angle $wlp = fbI = lbP$; *ergo* the Segments of the Curve Pl, $P\varpi$, are equal; *ergo* $l\varpi = bl$.

So if bz were drawn, making the Angle $fbz = Iaf$, and the Segmental applied to bz, as it was to lp; then would the Curve Side of the Segmental, cut fl in the Point a; and ſo of the reſt.

Corollary.

Hence bz cuts JL between J and p.

Demonſtration.

Let a perpendicular be ſuppoſed to be let fall from the Point of Interſection of bz, with JL to fl, cutting fl in u. Then $bu = fa = ba + au = fu + au$, *ergo* $fu = ba$; but becauſe $ba > fk$, *ergo* $fu > fk$.

$lw = bf = lb + ba + aw = fw + aw + ab$; *ergo* $lb = fw$, but becauſe $lb > ba$, *ergo* $fw > ba$, *ergo* $fw > fu$; *ergo* u falls between w and f: And becauſe $fu > fk$, *ergo* u falls beyond k. *Ergo* u falls between w and k; and conſequently the Interſection of bz, with JL, falls between p and J.

Scholium.

Produce lp, cutting kx in x; then if you draw bx, ax; bx will cut JL between p and J; and ax will cut JL between J, and the Point of Interſection of bx with JL, and by reaſon of the ſmall Length of Jx, which might eaſily be ſhewn, the Lines bx, ax, will cut JL ſo as to make the Angles fbx, fax, ſo nearly equal to Iaf, Ikf, reſpectively, as that they may be eſteemed equal, in the Application of the Method drawn from hence to Mechanical Conſtructions.

Lemma II. *Problem.*

Let the Eye be at I, and let the Length of Il be given, together with the Angle Ilk, leſs than a Right Angle, and let the meaſured Length of lk be given.

Then let it be required to find the local Length of lk.

1. Produce kl and Il, and applying the Segmental at l, as in the Scheme, mark the Point y where the Curve Side of the Segmental cuts kl, and draw yx, ſo as that the Angle $kyx = Ilk$.

2. Then let fall If, perpendicular to lk produced, and taking $fk = 1$ Inch; erect at k, kx, perpendicular to fl, and cutting yx in x, as in the Scheme.

Then draw lx, and applying the Segmental at l, (as in the Scheme) let the Curve Side of it cut lf in b; ſo will you have lb appearing equal to One Inch.

Then draw bx, and applying the Segmental at b to bx, (as in the Scheme) let the Curve Side of it cut fl in a; ſo you will have ba appearing equal to another Inch, and ſo proceed to find the reſt of the required Diviſions; and then meaſuring the whole Length of kl, by the help of a good Scale, you may determine the local Length of it nicely enough.

Demonſtration.

By the Scholium, the Angle $flx = Ibf$, *ergo* as may be gathered from *Lemma* I. lb is rightly determined; and ſo of the reſt.

Scholium

Scholium I.

Becauſe the Lines *Il*, *lf*, *&c.* are too Long (for the moſt part) to be drawn Geometrically in their full Length; and in caſe of contracting them by a Scale, the Diviſions *lb*, *ba*, *&c.* would be but uncertainly found: Therefore proceed for the *Second Lemma*, by Calculation thus:

Fig. 1.

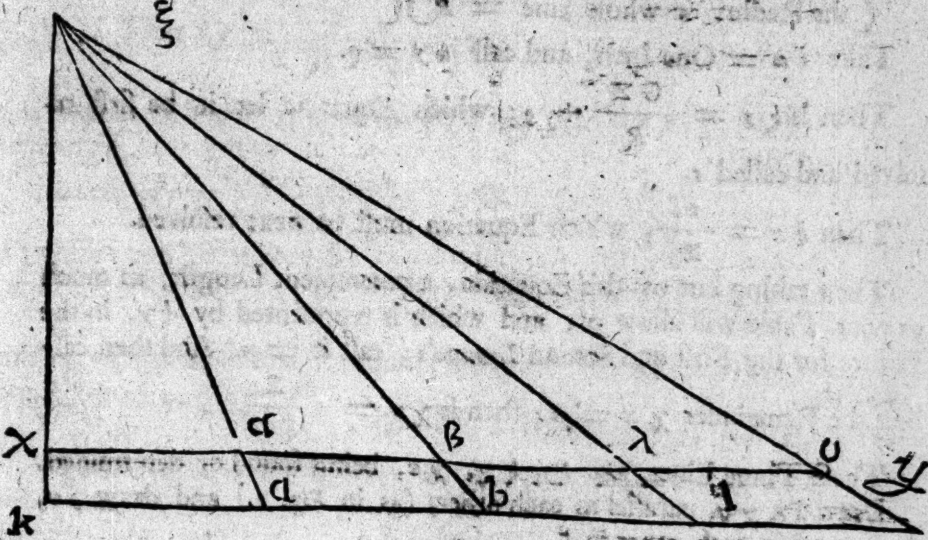
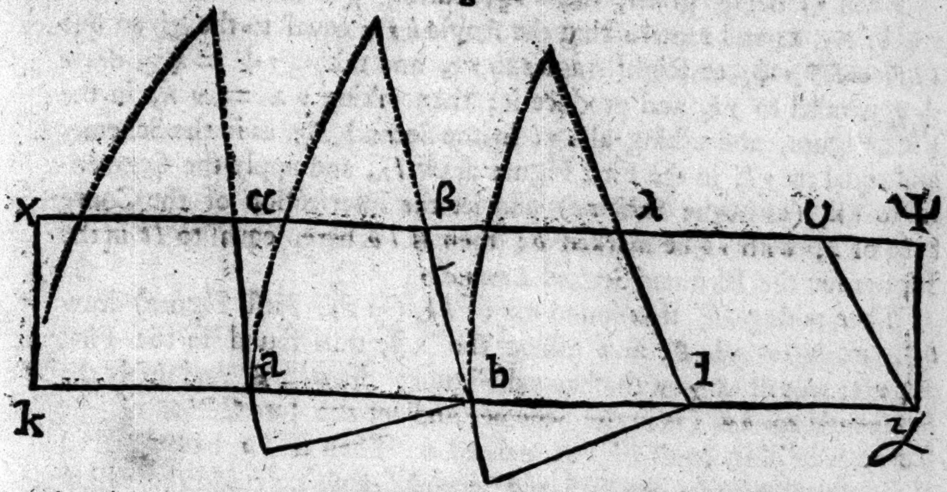


Fig. 2.



Let $\chi\upsilon$ be drawn in the Figure for the Firſt and Second *Lemma's*, parallel to ky, as in the Scheme there. (*See the Figure of the Firſt and Second Lemma's, Pag.* 48.

$$\text{Call}\left\{\begin{array}{l} Il = G \\ \text{the Sine of the Angle } Ilf = S \\ \text{the Sine of the Angle } lIf = \Sigma \\ \text{the Radius or whole Sine} = R \end{array}\right\}\text{These are all Given.}$$

Take $l\omega$ = One Inch, and call $\omega y = \varrho$.

Then is $ky = \frac{G\Sigma}{R} + \varrho$; which Equation let it be firſt reſolved and called ε.

Then $kx = \frac{\varepsilon S}{\Sigma}$, which Equation muſt be next reſolved.

Then taking out of this Equation, a convenient Length, as much as your Table will allow of, and which is repreſented by $k\chi$, in the Figure for the Firſt and Second *Lemma's*; call it $= n$. And then calling the Remainder $\chi x = m$; then is $\chi\upsilon = \frac{m\Sigma}{S}$.

Theſe Three Lines, *viz.* ky, $k\chi$, $\chi\upsilon$, being found or determined. Draw ky, $\chi\upsilon$, parallel to each other, (as in *Fig.* 1.) and draw $y\upsilon$, $k\chi$, cutting each other in ξ.

Then yl being given, draw $l\xi$, cutting $\chi\upsilon$ in λ. Then draw yk in *Fig.* 2. and conſtructing the Angle $ky\upsilon$ equal to the given one Ilf, erect $y\psi$, at Right Angles to yk, and taking $y\psi = k\chi$, draw $\psi\upsilon$ parallel to yk, and produce it; then taking $\upsilon\lambda = \upsilon\lambda$, in the Firſt Figure, and taking alſo yl in the Second *Fig.* as in the Scheme, and equal to yl; in the Firſt Figure draw $l\lambda$, and apply the Segmental to $l\lambda$, (as in the Scheme) and let the Interſection of the Curve Side of it, with yk be marked b; then is lb here, equal to lb in the Figure for the Firſt and Second *Lemma's*.

Then taking lb, thus found out of lk, (in the Firſt Figure) draw $b\xi$, cutting $\upsilon\chi$ in β; and taking the $\lambda\beta$, thus found in the Firſt Figure, out of $\lambda\chi$ in the Second Figure; draw $b\beta$, and apply the Segmental to $b\beta$, (as in the Scheme) and let the Interſection of the Curve Side of it, with yk, be marked a. Then is ba, here equal to ba in the Figure for the Firſt and Second *Lemma's*. And ſo repeating this Proceſs, you will find all the Diviſions.

1 *Note*, If yk be too long for your Table, you may begin with part of it, and proceed after the Method in the Note following the

Lemma foregoing, *Prob.* IX. accommodating thereto the Method deſcribed; or you may work after the Method in the Second *Scholium* following.

Demonſtration of the Scholium.

1. That $ky = \frac{G\Sigma}{R} + \varrho$. $R : ll\,(G) :: \Sigma : fl$; *ergo* $fl = \frac{G\Sigma}{R}$ but becauſe $l\omega = 1$ Inch $= fk$; *ergo* $k\omega = fl = \frac{G\Sigma}{R}$. And becauſe $\omega y = \varrho$; *ergo* $ky = k\omega + \omega y = \frac{G\Sigma}{R} + \varrho$.

2. That $kx = \frac{tS}{\Sigma}$. $R : Ik\,(G) :: S : If$; *ergo* $If = k\mathcal{J} = \frac{GS}{R}$. Let a Line $\omega\mathcal{J}$, be ſuppoſed to be drawn. Then the Angle $k\omega\mathcal{J} = Ilf = kyx$; *ergo* the Triangles $k\omega\mathcal{J}$, kyx, are equiangular; *ergo* $k\omega : k\mathcal{J} :: ky : kx$; that is, calling $ky = t$. $\frac{G\Sigma}{R} : \frac{GS}{R} :: t : kx$; *ergo* $kx = \frac{tS}{\Sigma}$.

3. That $\chi v, = \frac{m\Sigma}{S}$. $kx : ky :: \chi x : \chi v$, that is, calling $\chi x = m$. $\frac{tS}{\Sigma} : t :: m : \frac{m\Sigma}{S}$, *ergo* $\chi v = \frac{m\Sigma}{S}$.

4. That the Diviſions are rightly determined. In the Figure for the *Lemma's*, produce χv, and erect $y\psi$ perpendicular to ky, and cutting χv in ψ; *ergo* $y\psi = k\chi$.

In the Firſt Figure for the *Scholium*, $yl : v\lambda :: yk : v\chi$. And becauſe the Angle kyv, in the Second Figure, $= kyx$ in the *Fig.* for the *Lemma's*, as alſo the Angle $ky\psi$, and the Lines $y\psi$, $yl =$ in both reſpectively; becauſe alſo the Line $\psi\chi$, here parallel to yk, as there: *Ergo* $v\lambda$ here, $= v\lambda$ there; and conſequently $l\lambda$, here is a part of lx there; and the Angle $kl\lambda$, here $= klx$ there: *Ergo* lb, is determined by the Segmental here, as there: And ſo of the reſt.

Scholium II.

In the Figure for the *Lemma's*, if the Length of *l k*, together with the Angle *l k l*, (it being not leſs than a Right Angle) and the meaſured Length of *k y* were given; then might the First *Problem* be performed more eaſily thus, without drawing *l k* in its full Length.

Fig. 1.

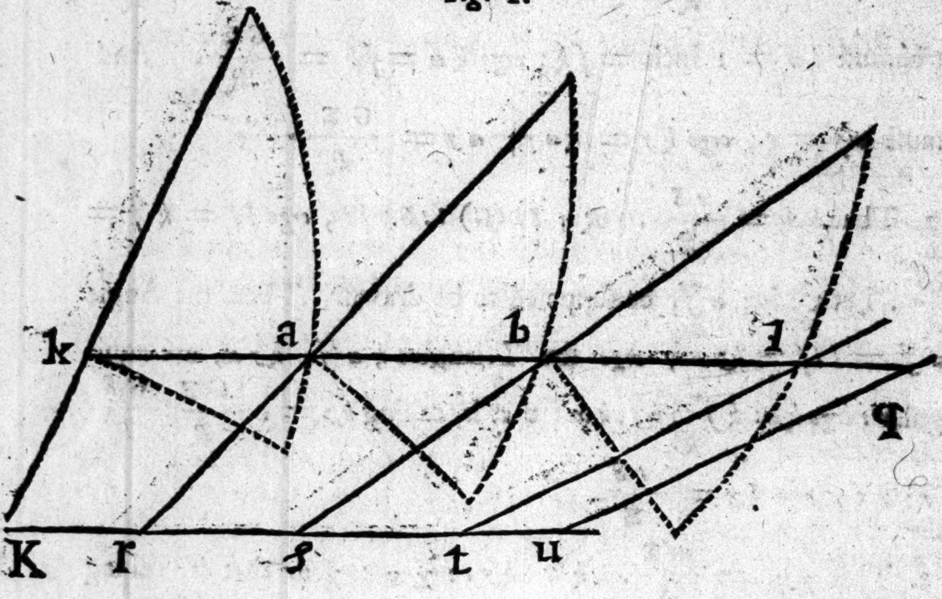
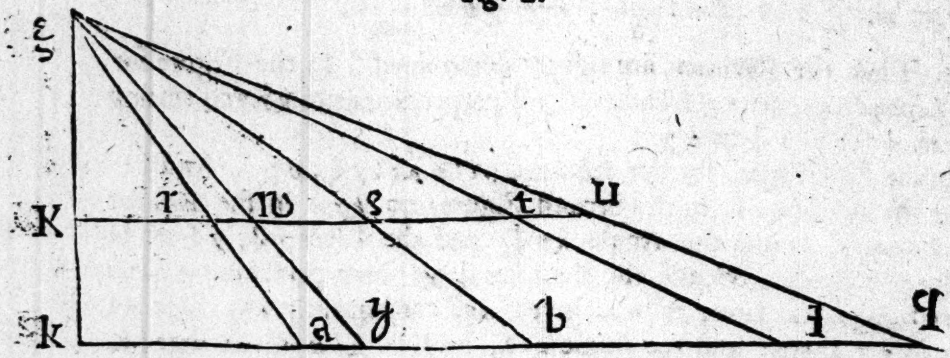


Fig. 2.



Let any Arbitrary Length be ſuppoſed to be taken out of the Line *ky*, (in the Figure for the *Lemma's*, *viz*. *kq*, and let *lq* be ſuppos'd to be drawn; as alſo *K u* parallel to *kq*, and cutting *lq* in *u*.

Call $kq = t$. Call $kl = G$. Call $kK = n$, and $Kl = m$: Theſe are all given Quantities.

Then becauſe $kl : kq :: Kl : Ku$; i. e. $G : t :: m : Ku$. *Ergo* $Ku = \frac{tm}{G}$.

Take then ſome Part of lk, *viz*. ſo much as your Table will admit of for n, and ſo you have m given, *viz*. $G - n$; and then reſolving the Equation $\frac{tm}{G}$, you have Ku.

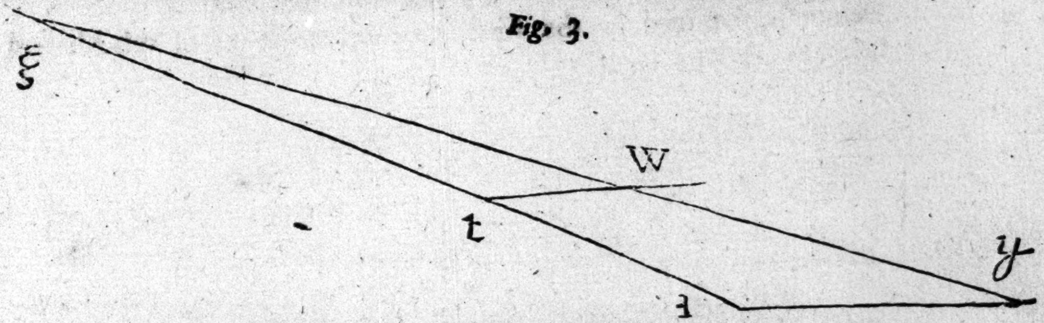
This Line being found, draw kq, Ku parallel to each other, as in *Fig*. 1. and draw qu, kK interſecting each other in ξ. Then in another Scheme, *viz*. in *Fig*. 2. draw $Kk = n$, and produce it; and at k and K draw parallel Lines ſo as that the Angle $Kkq = lkl$ in the Figure for the Lemma's. Then applying the Segmental at k to Kk, produced as in the Scheme, mark the Point of Interſection of the Curve-ſide of the Segmental with kq, with the Letter a; ſo you have ka here $= ka$, in the *Fig*. for Lemma's.

Then taking ka, thus found in the Second Figure, out of kq in the Firſt Figure, draw $a\xi$ cutting Ku in r.

Then taking Kr, thus found in *Fig*. 1. out of Ku, in the Second, draw ra and produce it; and then applying the Segmental at a, to ra, (produced as in the Scheme) mark the Point of Interſection of the Curve Side of the Segmental with kq, with the Letter b; ſo you have ab, here $= ab$, in the *Fig*. for the *Lemma's*. And ſo repeating this Proceſs, you will find all the Diviſions between k and q.

Then as for the other Diviſions beyond q; for ſuch there will always be when ky is of a conſiderable Length; becauſe kq muſt be ſo ſhort as to be comprehended within your Table agreeably. As for the other Diviſions, having proceeded ſo far as kl, begin again thus:

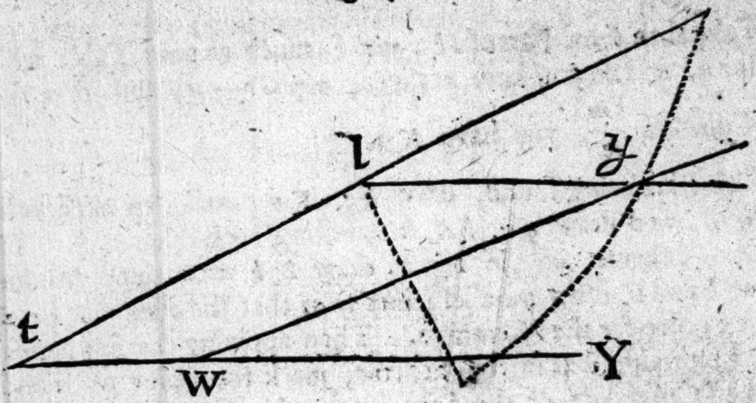
Fig. 3.



In the 4th *Fig* draw tw, tl, making an Angle $= ltu$ in the 2d. *Fig*. then draw ly parallel to tw, and producing tl, applythe Segmental

mental at l to $t\,l$, produced as in the Scheme; then mark the Point of Interſection of the Curve Side of the Segmental with ly with the Letter y, ſo you have ly here $= ly$, in the *Fig.* for the *Lemma's.*

Fig. 4.



And then if there are more Diviſions. In the Third *Fig.* draw $\xi t l$ (as in the Firſt *Fig.* making ξt here, $= \xi t$ there, and $t\,l$ here, $= tl$ there, and draw $t\,w$, ly, parallel.

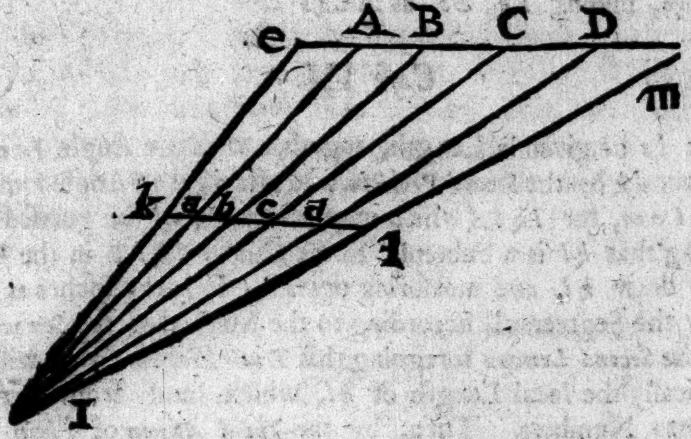
And taking ly in this Third *Fig.* $= ly$; in the 4th *Fig.* draw $y\xi$ cutting $t\,w$ in w; or inſtead of drawing the Third *Fig.* you may take $ky = ly$ out of kq in the Firſt *Fig.* and drawing $y\xi$, that will cut Ku in w, making $kw = tw$ in the Third Figure. And then taking $t\,w$ (found thus in the Third *Fig.*) out of tY in *Fig.* IV. draw wy and produce it. And then applying the Segmental at y to wy produced, proceed as before.

I ſuppoſe there is no need of a Demonſtration of this Method; the Reaſon being given why $Ku = \frac{tm}{G}$, and the application of the Demonſtration uſed in the Firſt Scholium to the reſt of this Method being ſo Obvious.

Problem

Problem X.

Let the Eye be at *I*, and let the Line *e m* be ſeen as the Line *k l*, the Angle *I e m* being not leſs than a Right-angle; and let the meaſured Length of *k l* be given.



Then let it be required to divide *e m* into *e A*, *A B*, *B C*, *&c.* ſo as that each of thoſe Diviſions ſhall appear equal to one Inch in Length.

Caſe I.

Let *I k* be given in Length, together with the Angle *I k l*.

Then meaſuring upon *k l* ſo many Inches as is required by the Segmental, according to the Method of the *Second Scholium* of the *Second Lemma* foregoing this *Tenth Problem*, you will have Geometrically the local Length of *k l*, which muſt from thence be reduced into Numbers.

Then, by the *Third* and *Second Axioms* of *Plain Trigonometry* you will have *I l*; and then, by the *Second Scholium* of *Prob.* IX. you will have the Diviſions *e A*, *A B*, *B C*, *&c.*

Caſe II.

Let *I l* be given in Length, together with the Angle *I l k*.

Then meaſuring upon *l k* ſo many Inches as is required by the Segmental, according to the Method of the *Firſt Scholium* of the *Second Lemma* foregoing this *Tenth Problem*, &c. as in *Caſe* 1. Then, as in *Firſt Caſe*, finding *I k*, &c. as in *Caſe* 1.

Caſe III.

Let *I e* be given in Length, together with the Angle *I e m*.

Find out *I k* by the *Second Problem*, and aſſuming an Angle ſomething leſs than *I e m*, for *I k l* (which may pretty nearly be gueſſed at by conſidering that *k l* is a Subtenſe to the Curve *f k l T* in the *Fig.* for *Prob.* 3.) draw *k l*, and meaſuring upon *k l* ſo many Inches as is required by the Segmental, according to the Method of the *Second Scholium* of the *Second Lemma* foregoing this *Tenth Problem*, you will have Geometrically the local Length of *k l*, which muſt from thence be reduced into Numbers. Then, by the *Third Axiom* of *Plain Trigonometry*, you will have the Angle *e I m*; and then, by the *Second Axiom* of *Plain Trigonometry*, you will have *I m* and *e m*.

So then, having *I e*, *I m*, *e m*, find out the New *k l* by the *Sixth Problem*, and the Diviſions *e A*, *A B*, *B C*, &c. by the *Second Scholium* of the *Ninth Problem*. And if *e m* thus aſſumed be too long or too ſhort to ſatisfy the Demand, you may ſhorten or lengthen it accordingly after the Method of the *Firſt Scholium* of the *Ninth Problem*.

Caſe IV.

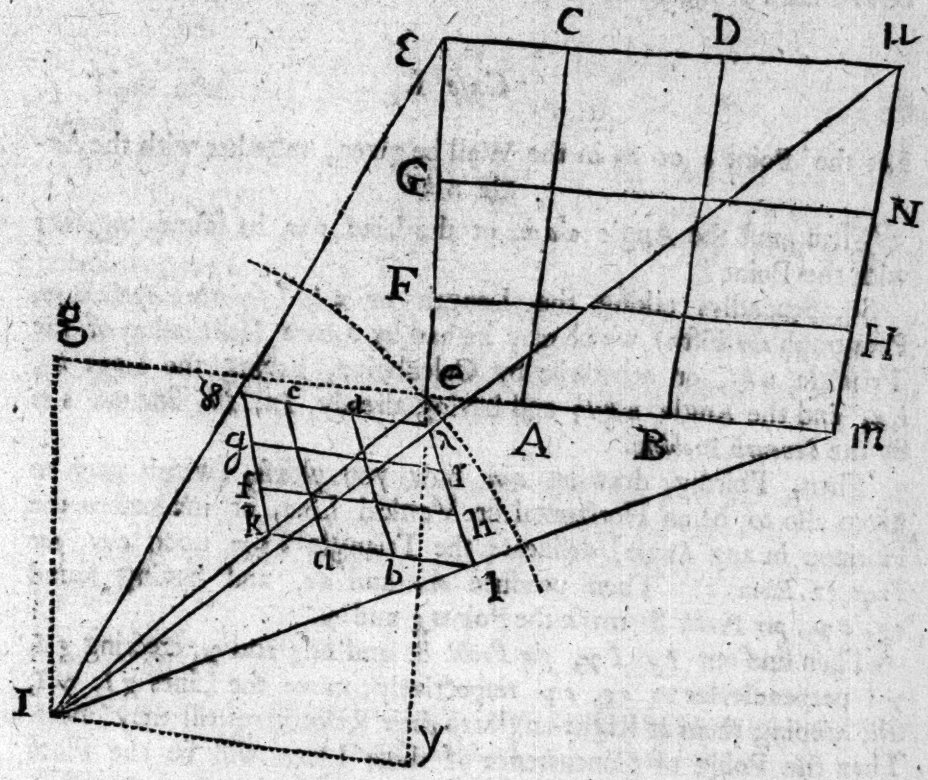
Let *I m* be given in Length, together with the Angle *I m e*.

Find out *I l* by the *Second Problem*; and aſſuming an Angle ſomething bigger than *I m e* for *T l k*, (which may be pretty nearly gueſſed at, conſidering that *l k* is a Subtenſe to the Curve *T l k f* in the Figure for the *Third Problem*) draw *l k*, and meaſuring upon *l k* ſo many Inches as is required by the Segmental, according to the Methed of the *Firſt Scholium* of the *Second Lemma* foregoing this *Tenth Problem*, you will have Geometrically the local Length of *l k*, &c. as in the *Third Caſe*; only inſtead of finding out *I m*, *e m*, you muſt find *Ii*, *em*; and *lk* muſt be lengthened or ſhortened at the End marked with *k*.

The Reaſon of this Method is ſo obvious that I omit any farther Demonſtration.

Problem XI.

Let the Eye be at *I*, and let the four-ſided Figure (or Trapezium) *knλl* be given; that is to ſay, let the meaſured Lengths of *kl* and *kn* be given, and every Side of each little four-ſided Figure (or Trapezium) made by the croſſing of the Lines *ac*, *bd* with *fh*, *gn*, appear equal to one Inch in Length after the Method of the *Firſt Problem*.



Then let it be required to delineate againſt a Wall the Figure *eεμm*, or to conſtruct a Frame *eεμm*, divided into Trapezia's, which ſhall exhibit to the Eye at I a chequer'd Parallelogram divided into little Parallelograms, whoſe Sides ſhall appear equal to an Inch in Length.

Firſt, Find out each Side, *viz.* em, $e\varepsilon$, $\varepsilon\mu$, μm, together with their correſpondent Diviſions ſeverally, by the *Tenth Problem*, according as each Caſe doth require.

> Note, *When the Angles* Ikl, $Ik\kappa$ *are given or found, you muſt determine the Angles* $I\kappa\lambda$, $Il\lambda$ *by Geometrical Rules, becauſe* κl *is a Plane; and then meaſuring by the Segmental,* $\kappa\lambda$ *and* $l\lambda$ *muſt be found.* (See the Application of this Method at the End of the *Third Paragraph.*)

Then, as for the reſt of the Operations, I ſhall conſider them under ſeveral Heads or Caſes, to which, by reaſon of the Variety of certain *Data* or *Aſſumpta*, they will be referrable.

Caſe I.

Let the Point e or m in the Wall be given, together with the Angle κkl.

Then muſt the Angle εem, or the Line εm, be found together with the Point I.

So, Secondly, taking the Length of κl, (*ſee the Application,* Paragraph *the* Fifth) which may be had by a bare Delineation of the Triangle κkl, or otherwiſe by Calculation, having the Legs kl, $k\kappa$, and the Angle κkl; and having already $I\kappa$, Il, find out εm by the *Seventh Problem.*

Then, Thirdly, drawing em how you pleaſe, (which may be given alſo to be an Horizontal or Vertical Line, or inclined to the Horizon in any Angle) delineate the Triangle $e\varepsilon m$ upon em, *per Prop.* 22. *Elem.* 1. Then produce me and εe, and having found eg, $e\gamma$, *per Probl.* 8. mark the Points g and γ.

Then find out Ig, $I\gamma$, *per Probl.* 8. and at g and γ, erecting gI, γI perpendicular to eg, $e\gamma$ reſpectively, move the Lines gI, γI, ſtill keeping them at Right-angles to their Reſpectives till they meet. Then the Point of Concurrence of thoſe Lines will be the Place of I.

Fourthly, Conſtruct upon *εm* the Triangle *εμm*; and draw *AC*, *BD*, *FH*, *GN* through the Points *A*, *B*, *C*, *D*, which were found in the Firſt Paragraph.

So you have the Figure *eεμm* divided into Trapezia's by the Groſs-lines *AC*, *FH*, &c. which will exhibit to the Eye at *I* a Parallelogram, Two of whoſe Angles are equal to *κkl* each; which Parallelogram alſo is divided into little Parallelograms, correſponding with the ſeveral Trapezia's in the Figure *eεμm*, each of whoſe Sides appear equal to One Inch.

Note, *If the Horizontal Heighth of the Eye be given, or the Vertical Diſtance, viz.* γ; *then* γe *being found by the* Eighth Problem, e *muſt be determined from* γ; *and then* eε *muſt be taken out of* γe *produced, and then proceed to erect the Triangle* εme *upon* εe, &c.

Caſe II.

Let the Points *I* and *e* be given.

Then muſt the Angles *εem*, *κkl*, or the Lines *εm*, *κl* be found.

So, Secondly, draw the given Line *Ie*, and conſtruct the Angles *Ieγ*, *Ieg*, given or found by the *Eighth Problem*. Then producing *γe*, *ge*, take *em*, *eε*; ſo drawing *εm*, you have *εm* by meaſuring the Length of it, and the Triangle *εme*.

Then having *Iε*, *Im* already, find out *κl* by the *Sixth Problem*; ſo you have *κl* alſo, and the Triangle *κlk*.

Then, Thirdly, proceed as in the *Fourth Paragraph* of the *Firſt Caſe* for the reſt.

Note, *If the Point* m *be given, draw* Im, *and conſtruct the given Angle* Img; *and taking* me *out of* mg, *you have the Point* e. *And then draw* Ie, *and conſtruct the Angle* Ieγ, &c. *as before.*

Caſe III.

Let the Point *I* be given, together with the Poſition of *em*, with reſpect to an Horizontal or Vertical Line.

Then muſt the Angles *εem*, *κkl*, or the Lines *εm*, *κl* be found.

So, Secondly, with the Line Ie, as with a Radius, from the Center I, deſcribe an Arch of a Circle, moving a Ruler, eg, along with Ie in the Situation given, till the Angle Ieg be equal to the given one found by the *Eighth Problem.*

Then producing ge, and conſtructing the Angle $Ie\gamma$, found by the *Eighth Problem*, produce γe alſo.

And then taking $e\varepsilon$, em, draw εm: So by meaſuring you have the Length of εm in Inches, and the Triangle $\varepsilon e m$.

And having $I\varepsilon$, Im already, find out κl by the *Sixth Problem*: So you have the Triangle κkl.

Then, Thirdly, proceed as in the *Fourth Paragraph* of the *Firſt Caſe* for the reſt.

Demonſtration of the Firſt Caſe.

The Angles Iem, $Ie\varepsilon$ are rightly determined, and the Lines em, $e\varepsilon$ are taken of a juſt Length to exhibit kl, $k\kappa$ reſpectively, in the Angles Ikl, $Ik\kappa$, and of the Lengths required by the Conſtruction; and becauſe Ig, $I\gamma$, eg, $e\gamma$, and the Angles Ige, $I\gamma e$, are conſtituted according to the *Eighth Problem.*

And the Lines em, $e\varepsilon$ are rightly divided to exhibit the Diviſions of kl, $k\kappa$ reſpectively, each of which do appear equal to One Inch, as is plain by the Conſtructions of them.

Alſo the Angle κkl is rightly determined, becauſe not only kl, $k\kappa$, the Legs of the Angle κkl, are duly repreſented by em, $e\varepsilon$, but alſo the Subtenſe κl is rightly exhibited by εm, according to the *Seventh Problem*; and conſequently the whole Triangle κkl is rightly repreſented by the Triangle $\varepsilon e m$.

For like Reaſons, if the Triangle $\varepsilon\mu m$ were turned upon its Baſis εm, till the Line $l\mu$ were of the Length required; then would the Triangle $\varepsilon\mu m$ be rightly conſtituted to repreſent the Triangle $\kappa\lambda l$; and the Diviſions εC, CD, &c. would repreſent κc, cd, &c.

But here, becauſe the Trapezium $e\varepsilon\mu m$ is a Plane, and therefore, in ſtrictneſs, the Line $l\mu$ is too ſhort to repreſent $l\lambda$, *per Caſe* 2. of *Theor.* 2. yet is the Difference ſo inconſiderable in the Triangle $\varepsilon\mu m$, whoſe perpendicular Heighth μ, from the Baſis εm, is only of ſome few Feet in Length, that it is not to be heeded.

And ſo the Triangles $e\varepsilon m$ and $\varepsilon\mu m$ being duly conſtituted, and the Sides rightly divided, the Croſs-lines AC, BD, FH, GN, will determine the Trapeziums rightly, which will be ſeen as Parallelograms required.

I ſhall not give a Demonſtration of the other Caſes, the expert Geometer being able from this to ſee the Reaſon of the ſeveral Proceſſes adapted to them.

Many other Caſes might be laid down, which upon Occaſion may be made uſe of: But thoſe who have need of ſuch, may eaſily, with a little Inſight into Geometry, ſupply the Want of them here to ſerve their Purpoſe.

Corollary.

Hence you have a Method of drawing Pictures, or carving Statues to be ſeen at a Diſtance obliquely in any given Angle, or otherwiſe; which ſhall appear in the Bigneſs and Situation required.

Namely, By aſſuming or finding the Four-ſided Figure *kκλl*, divided into little Four-ſided Figures, whoſe meaſured Lengths of the Sides *kl*, *kκ*, are given equal to the Length and Breadth of the Picture or Statue reſpectively; and whoſe Angles *κkl*, *κλl*, are given or found according to the ſeveral Caſes in this *Eleventh Problem.*

And then inſcribing the Picture or Statue in a Parallelogram divided into little Parallelograms, whoſe Sides repreſented by *kl*, *kκ*, are equal to the meaſured Length of thoſe Sides reſpectively; and whoſe Angles, repreſented by the Angles *κkl*, *κλl*, are equal to thoſe given or found ones.

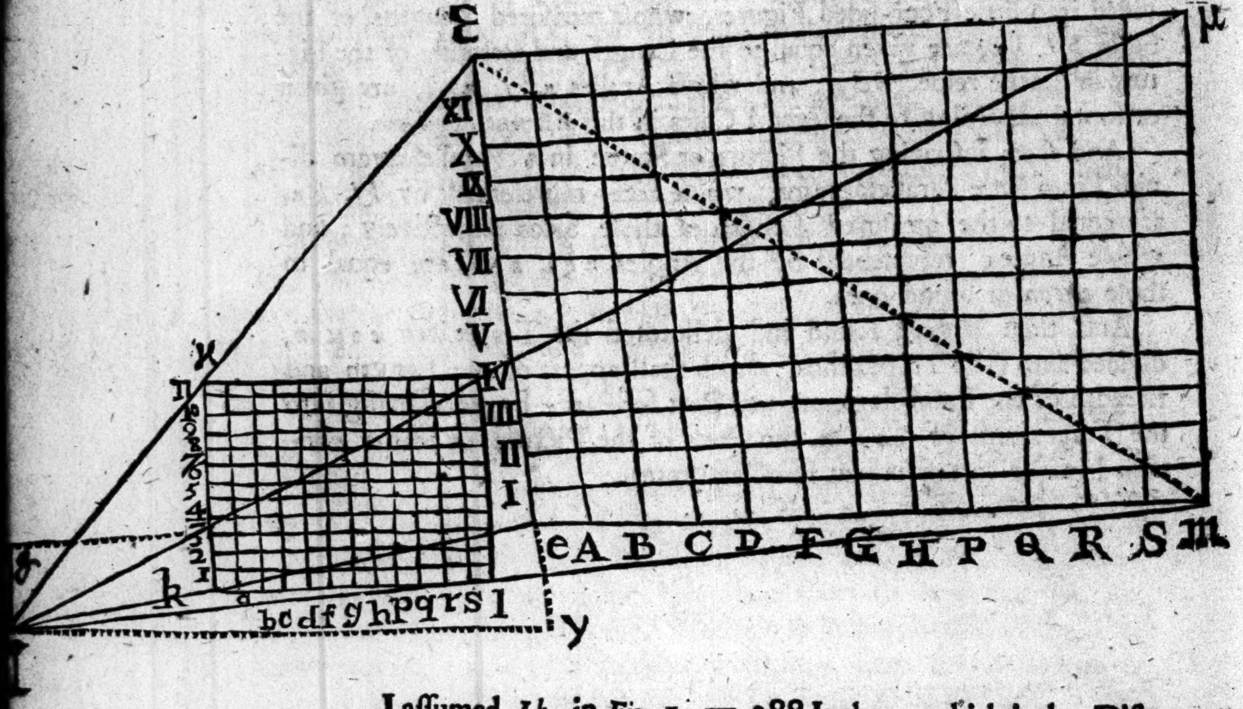
And then having found and delineated the Trapezium *eεμm*, divided into little Trapeziums, which ſhall appear of the Length and Breadth of the Parallelogram, and Part for Part; by transferring into the Trapeziums of *eεμm*, the Parts of the Picture or Statue contained in the correſpondent Parallelograms.

The

The Application.

FOR an Illuſtration of this Method, I have here ſubjoined an Application of a particular Caſe.

Fig. 1.



I aſſumed *Ik*, in *Fig.* 1. = 288 Inches; which is the Diſtance I would have the Picture to be ſeen at.

Then I took the Angle $1kl = 160^\circ$, and the Angle $1kx =$ a Right-angle; which Two Angles do give the Situation I would have the Picture appear in.

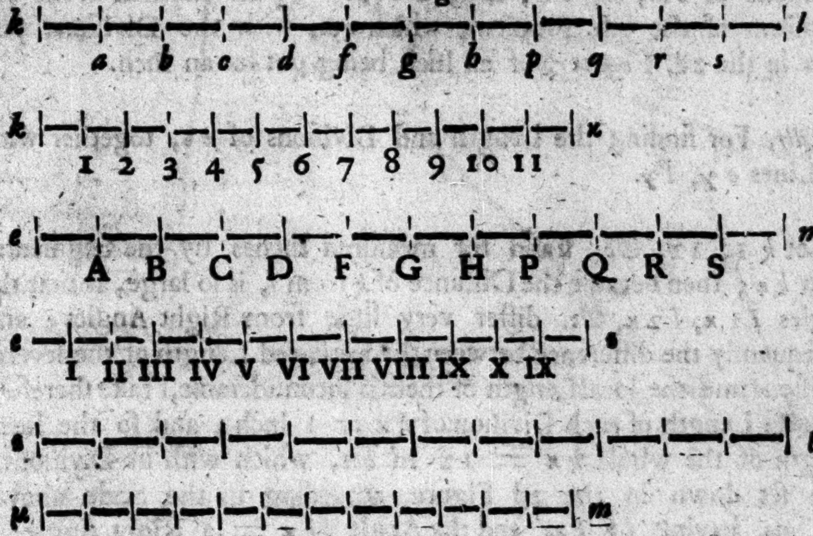
Alſo I aſſumed the meaſured Length of $kl = 12$ Inches, and the meaſured Length of $kx = 12$ Inches; which Two Lines, including the Breadth and Length of the Picture I would repreſent, give the Size of the Picture.

Firſt then, For finding the Length and Diviſions of em, together with the Lines eg, lg.

According to the Preſcription in the *Second Schol.* of *Lem.* 2. foregoing the *Tenth Problem*, I aſſumed $Kk = 8$ Inches, and $kq = 5$ Inches, and by reſolving the Equation $\frac{sm}{G}$ as there, I found $Ku = 4,8611$, &c.

Next, according to the Method in the ſaid 2d. *Lemma* and 2d. *Scholium* of it, I meaſured by the Segmental 12 Inches upon k[illegible] marking each Diviſion with Letters as in the 1ſt. *Figure*; and then Meaſuring the whole length, it appeared equal to 16,2 Inches. And this is the Local length of kl, which with its Diviſions, taking every fourth part of an Inch for an Inch. *See in Fig.* 2.

Fig. 2.



Thus having Ik and kl and the Angle Ikl, by the third *Axiom* of *Plain Trigonometry* I find the Angle $kIl = 1^{\circ}, 2, 49''$. And the Angle $Ilk = 18^{\circ} : 57', 11''$ and then by the 2d. *Axiom* of *Plain Trigonometry* I find $Il = 303{,}2392$.

Then by the fourth Problem I find $Ie = 325, 4973$

And by the ſaid Problem I find $Im = 345{,}0981$

And ſo having Ie, Im and the Angle eIm or kIl, I find by the 3d. *Axiom* of *plain Trigonometry*, the Angle $Iem = 162^{\circ} : 7' : 5''$. And the Angle $Ime = 16^{\circ} : 50' : 6''$. And then according to the Note following the 7th Problem by the 2d. *Axiom* of *plain Trigonometry* I find $em = 20{,}5355$.

Then having the Angles Ieg and eIg, in the Right-angled Triangle Ige, I find by the firſt *Axiom* of *plain Trigonometry* $eg = 309{,}7729 +$ and $Ig = 99{,}9461 +$.

Then according to the Method of the 2d. *Scholium* of the 9th. *Problem*, taking $l\chi = 320$, and conſequently $\chi e = 5{,}4973 -$.

And by reſolving the Equations $\frac{rd}{g}$, $\frac{rD}{g}$, as there, I find $\chi\lambda = 18$ and $en = 18{,}30922$.

Then having divided en and $\chi\lambda$, according to the Method of the *Lemma* foregoing the 9th Problem and marked the Diviſions, I find the Diviſions of em, *viz.* eA, AB, BC, &c. by the Method in the 2d. Scholium of the 9th. Problem, which em, with the Diviſions you have in the 2d. Figure $\frac{1}{4}$ of an Inch being put for an Inch.

2dly, For finding the Length and Diviſions of ex, together with the Lines $e\gamma$, $I\gamma$.

Let k 1, 12, *&c.* ſtand for meaſured Inches by the Segmental upon kx; then becauſe the Diſtance of k from I, is ſo large, as that the Angles $I1x$, $I2x$, *&c.* differ very little from Right Angles; and conſequently the difference between the meaſured Length of the ſeveral Diviſions, and the local Length of them is inconſiderable, I take therefore the local Length of each Diviſion of $kx = 1$ Inch; and ſo the local Length of the whole $kx = 12$ Inches, which with its Diviſions I have ſet down in the 2d Figure, according to the Scale above.

Thus having Ik, kx, and the Angle $Ikx =$ a Right Angle; I find by the 1ſt *Ax* of plain Trig. the Angle $kIx = 2^{\circ}\ 23'\ 9''\ 30'''$; and the Angle $Ixk = 87^{\circ} - 36' - 50'' - 30'''$.

And then I find $I\kappa = 288, 24674$.

Then, by the 4th Problem, I find $I\iota = 325, 81249$.

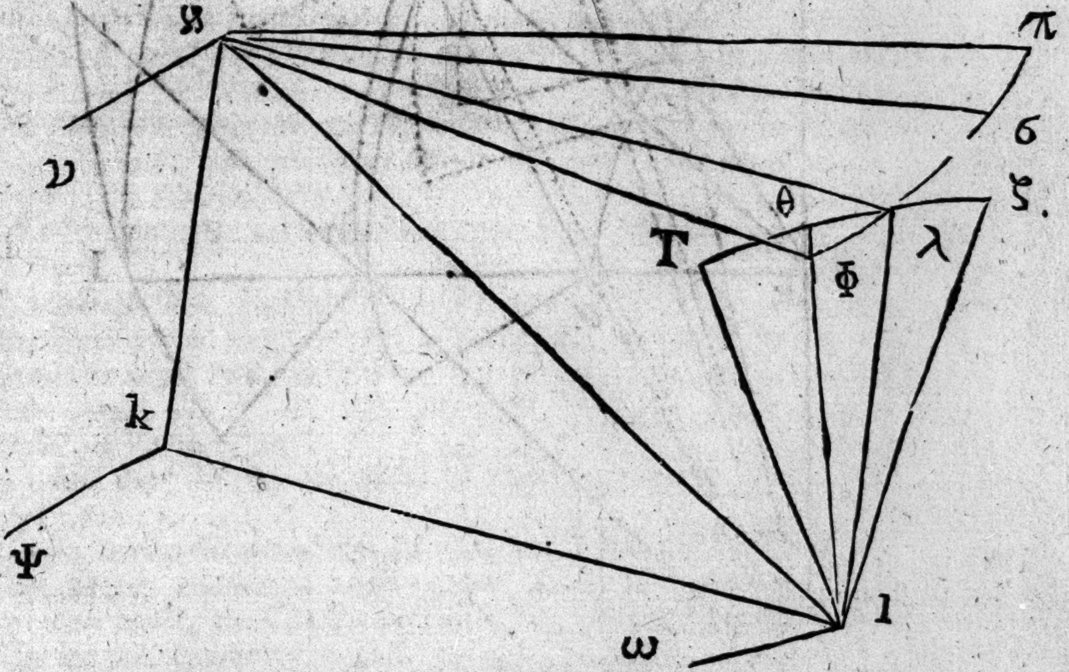
And ſo having Ie, $I\iota$, and the Angle $eI\iota$, or $kI\kappa$, I find by the 3d Axiom of plain Trig. the Angle $Ie\iota = 90^\circ, 8', 17'', 50'''$, and the Angle $I\iota e = 87^\circ, 28', 32'', 40'''$. And then by the Second Axiom of plain Trig. I find $e\iota = 13, 564$.

Then having the Angles $Ie\gamma$, $eI\gamma$, in the Rightangled Triangle $I\gamma e$, I find by the firſt Axiom of plain Trig. $e\gamma = 0, 7855$, and $I\gamma = 325, 4963$.

And in dividing $e\iota$ into its Parts; becauſe by reaſon of the ſmall Difference between the Angles $Ik\kappa$, $Ie\iota$, the Difference of the Parts will be inconſiderable. I only divide it into $12 =$ Parts.

3*dly*, For the finding the local Lengths of $\kappa\lambda$, $l\lambda$, and the Angles $I\kappa\lambda$, $Il\lambda$, which as I have obſerved in the Note following the 1ſt Paragraph of the 11th Problem, are determined, when the Angles Ikl, $Ik\kappa$ are given.

Fig. 3.

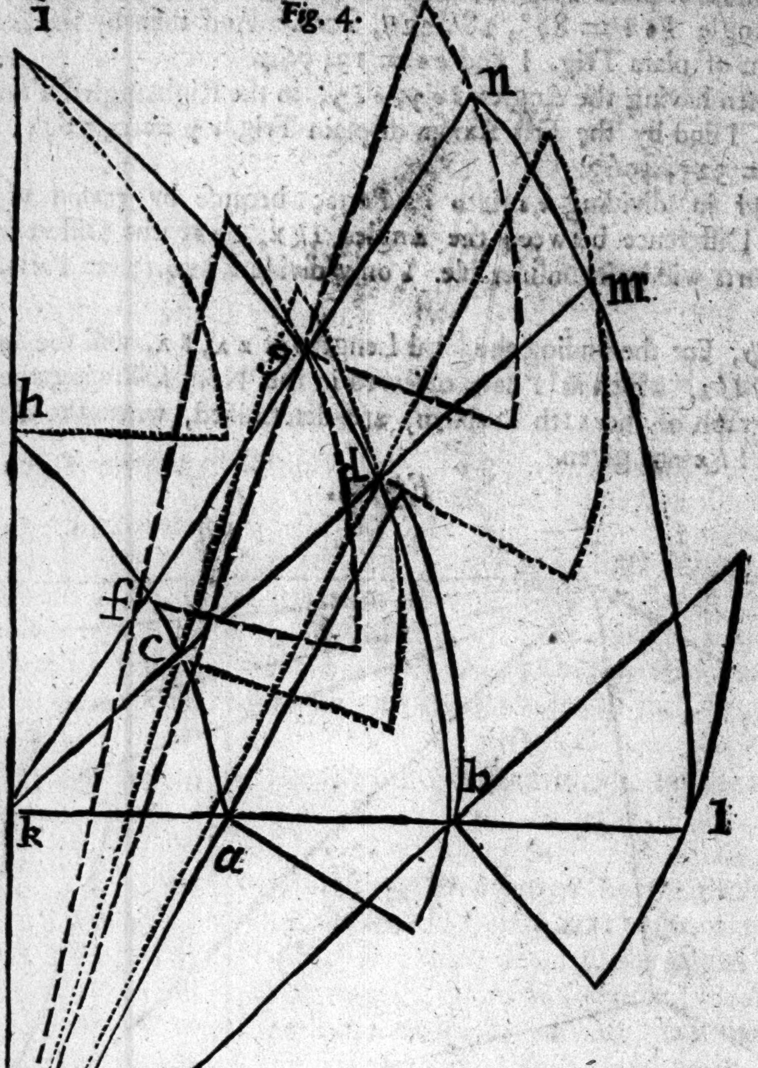


Conſtruct the Angles Ψkl, ωkl, and $\Psi k\kappa$, $\nu\kappa k = Ikl$, Ilk, and $Ik\kappa$, $I\kappa k$, reſpectively, as found above; kl, $k\kappa$, here being equal

kl, $k\kappa$, in the Firſt Figure; and place kl, $k\kappa$, in any Angle $\kappa k l$ againſt a plain Wall.

Then aſſuming arbitrarily, the Angles $\varpi\kappa\nu$, $\sigma\kappa\nu$, $\varphi\kappa\nu$, draw $\kappa\varpi$, $\kappa\sigma$, $\kappa\varphi$, and meaſure upon each of them by the Segmental, according to the Method of the 1ſt or 2d Scholium of the 2d Lemma fore-

Fig. 4.



going, the 10th Problem, as the Caſe requires, the number of Inches kl is given of; and draw the Curve $\varphi\sigma\pi$, through the Points $\varphi\,\sigma, \pi$, which terminate the local Lengths of $\kappa\varphi$, $\kappa\sigma$, $\kappa\pi$, reſpectively. Then every Line drawn from κ to the Curve will be of the meaſured Length of kl, as may be plain by conſidering the 4th Scheme, where the meaſured Length of every Line drawn from k, to the Curves $bdgi$, lmn, is equal to the meaſured Lengths of bk, kl reſpectively, as might be demonſtrated at large.

Then drawing $l\zeta$, $l\theta$, lT, upon the Wall, aſſuming arbitrarily the Angles $\zeta l\omega$, $\theta l\omega$, $Tl\omega$, meaſure by the Segmental as above upon each of them, the Number of Inches which $k\kappa$ is given of, and draw the Curve $\zeta\theta T$ dextrouſly through the Points ζ, θ, T.

Then every Line drawn from l to the Curve $\zeta\theta T$, will be of the meaſured length of $k\kappa$ as before: And then the point of Interſection of the two Curves, will be the place of λ; and ſo drawing $\kappa\lambda$, $l\lambda$, you will have the local length of them by meaſuring them; and meaſuring the Angles $\lambda\kappa\nu$, $\lambda l\omega$, you will have the Angles $I\kappa\lambda$, $Il\lambda$, reſpectively.

But in this Caſe I need not be at the trouble of drawing the Curves $\varphi\sigma\pi$, $T\theta\zeta$, becauſe ſuppoſing $\kappa\lambda$ equal and parallel to kl, and ſuppoſing $l\lambda$ equal and parallel to $k\kappa$, the demand would be ſo nearly ſatisfied thereby as to juſtify the Aſſumption of them; by reaſon that the Angle $I\kappa\lambda$, would be ſo nearly equal to Ikl, and the difference of length between $I\kappa$ and Ik, ſo ſmall, as that $\kappa\lambda$ may be eſteem'd parallel to kl, and for like reaſon $l\lambda$ may be eſteem'd equal to $k\kappa$; and ſo the Diviſions of kl, are applicable to $\kappa\lambda$, and the Diviſions of $k\kappa$ are applicable to $l\lambda$; and ſo too the Angle $I\kappa\lambda = Ikl = 160^\circ$.

Fourthly, For finding the Length and Diviſions of $\iota\mu$ and μm.

Having the length of $I\kappa$, as before, and taking the length $\kappa\lambda$, and the Angle $I\kappa\lambda$, as before, I find by the 3*d. Axiom* of *Plain Trig.* the Angle $\kappa I\lambda = 1^\circ, 2', 45'', 33'''$, and the Angle $I\lambda\kappa = 18^\circ, 57', 14'', 27'''$.

And then, by the 2*d. Axiom* of *Plain Trigonometry*, I find $I\lambda = 303{,}51825$.

So having the three Sides $I\kappa$, $I\lambda$, $\kappa\lambda$, I find the Length of $\iota\mu$, by the 7th *Problem* $= 20{,}5815 +$. *See the Proceſs following.*

And having the Lengths of $I\lambda$, Il, λl, I find the Length of μm, by the 7th. *Problem* $= 13{,}66$.

And there is no need of being at trouble to find the Diviſions of *e μ μ m*, they being ſo nearly determinable by the Diviſions of *e m*, *e* *See e μ*, *μ m*, with their Diviſions in *Fig.* 2d drawn according to the Scale above.

Then I aſſume the Angle *κ k l* to be ſeen as a right Angle.

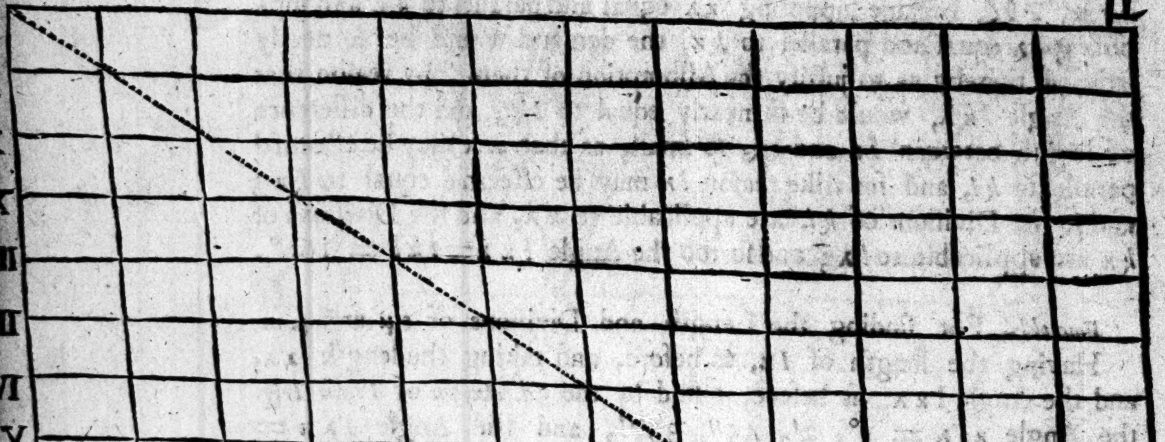
Fifthly, Therefore for finding *κ l* and *e m*, as the 1ſt. *Caſe* of the 11th. *Problem* directs.

By the 47th. *Prop.* of 1ſt. *Elem.* I find *κ l* = √ 406,44.

And by the 7th *Problem*, I find *e m* = 25,663.

Note, κ l, *may be found out more nicely by drawing a Curve of the Meaſur'd length of* κ l, *in the Square* k κ λ l, *in the 6th* Fig *ſomethings after the manner of finding* κ λ, l λ, *though with more difficulty, as thoſe who underſtand Geometry, may from the Method of finding* κ λ, l λ, *conceive. But for the moſt part the way I have taken here ma ſuffice.*

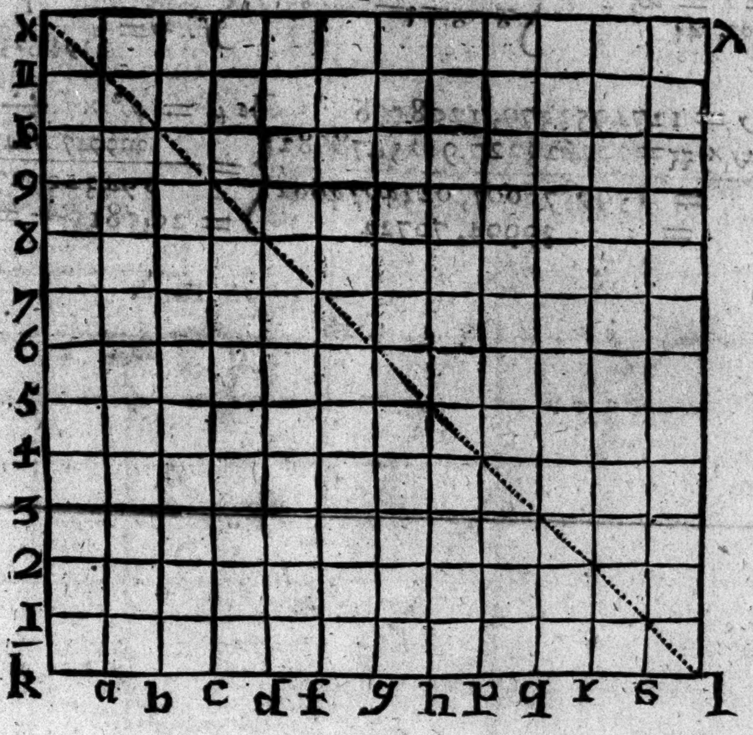
Fig. 5.



Sixthly, I delineate upon a board the Triangle *e ε m* (*See Fig.* 5th.) and upon *ε m*, I delineate the Triangle *ε μ m*, dividing the Sides *e m*, *e ε*, &c. from the Diviſions of thoſe Lines in the 2d. *Fig.* and then drawing the croſs Lines I have the Trapezium *e ε μ m*, divided into Trapezia's each of which will appear equal to an Inch Square, and as a Square, as is to be ſeen in the 5th *Fig.*

And then including the Picture in the Square *k κ λ l*, in the 6th *Fig.* who ſides are 12 Inches long, and which is divided into 144 equal

Fig. 6.



little Squares, and Transferring the parts of the Picture in the little Squares into the Correſpondent Trapezia's dextrouſly, I have a Picture which will appear to the Eye at *I*, as the Picture I would repre- in the Situation, at the Diſtance, and of the Size required.

Laſtly, Either aſſuming the heighth of the Eye, or the place of *e*, or *m*, the board may be placed againſt a Wall, and the Point for *I*, found after the Method of the 1ſt. *Caſe* of the 11th. *Problem.*

Note, *It will be convenient to Aſſign the Diſtance and Situation of your Picture ſo, as that the leaſt conſiderable parts of it may be ſeen under an Angle equal to* 1′ *at leaſt, for otherwiſe the parts will not be Diſtinctly ſeen.*

Here follows the proceſs by the 7th. Problem for finding $^{s}\mu$.

$$f = 2500. \qquad ff = 6250000.$$

$$\left.\begin{array}{l} I\kappa = g = 288,24674 \\ I\lambda = eg = 303,51825 \\ \kappa\lambda = r = 16,2 \\ rr\ 262,44 \end{array}\right\} \left\{\begin{array}{l} f - g = x = 2211,75326 \\ f - eg = y = 2196,48175 \\ eg - g = z = 15,27151 \end{array}\right\} \left\{\begin{array}{l} xy = 4858075,67109 \\ zz = 233,21902 \\ ff - xy = 1391924,32891 \\ f \div xy = \dfrac{1}{1943,23027} \end{array}\right.$$

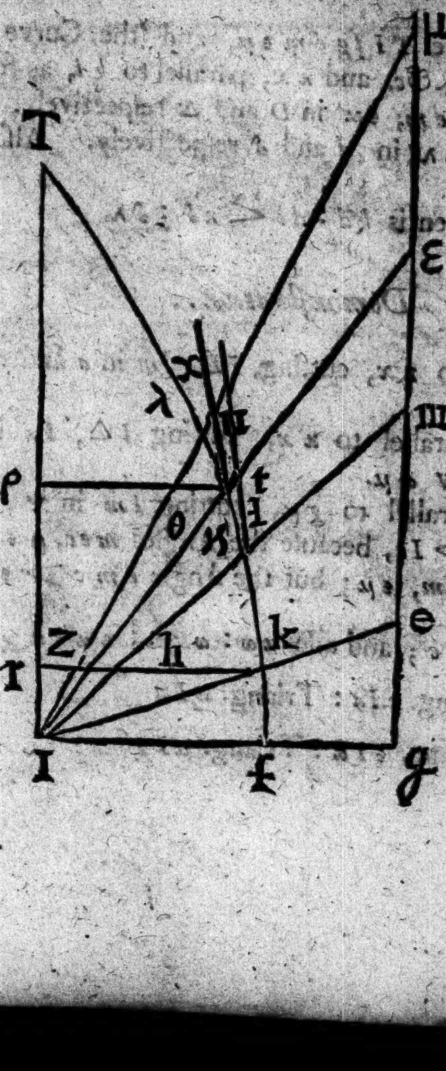
$$\left.\begin{array}{ll} rrxy = & 1274953379,1208596 \\ ff - xy \times zz = & 324623227,9025478682 \\ \hline \text{Sum} = & 1599576607,0234074682 \\ \sqrt{\ } = & 39994,70724 \end{array}\right\} \begin{array}{l} ^{s}\mu = \sqrt{^{s}} \times f \div xy. \\ = \dfrac{3999470724}{194323027} \\ = 20,5815+ = {}^{s}\mu. \end{array}$$

APPEN-

APPENDIX
TO
PERSPECTIVE.

Lemma I.

LET the Eye be at I, and let the Line $g\,e\,m\,\nu\,\mu$, infinitely continued, be ſeen as the Curve $f\,k\,l\,n\,\lambda\,T$, the Angle $I g m$ being a Right-angle; and taking any how out of $g\,\mu$, $e\,m$,



$\varepsilon\mu$, equal to each other, draw Ie, Im, $I\varepsilon$, $I\mu$ cutting the Curve in k, l, κ, λ reſpectively; and draw kr, $\kappa\rho$ parallel to Ig, cutting lm and $I\mu$ in b and θ reſpectively. Alſo kr cuts $I\mu$ in ζ.

Then is $bl > \theta\lambda$.

Demonſtration.

Draw the Subtenſes kl and $\kappa\lambda$, and produce kl cutting $I\varepsilon$ and $I\mu$ in t and u reſpectively. Draw alſo κx parallel to tu.

1 Becauſe $em = \varepsilon\mu$, *ergo* the Angle $eIm > \varepsilon I\mu$, *ergo* $tu < kl$, *ergo* much more $\kappa x < kl$.

2 The Angle $x\zeta k = x\theta\kappa$, but $> lbk$; *ergo* $\theta\kappa < bk$.

3 And becauſe the Angles $\theta\kappa x$ and bkl are equal, *ergo* the Subtenſe $bl > \theta x$, *ergo* much more $bl > \theta\lambda$.

Lemma II.

Conſtruct the Figure $TIfgem\varepsilon\mu$, and the Curve $fkl\kappa\lambda T$, drawing the Lines Ike, *&c.* and κx, parallel to kl, as for the *Firſt Lemma*; and biſecting em, $\varepsilon\mu$ in D and Δ reſpectively, draw ID, $I\Delta$, cutting kl and $\kappa\lambda$ in d and δ reſpectively. Alſo $I\Delta$ cuts κx in ω.

Then is $kd : dl < \kappa\delta : \delta\lambda$.

Demonſtration.

Draw ec parallel to κx, cutting ID, Im in a and c reſpectively; and draw am.

Then draw $\varepsilon\chi$ parallel to κx, cutting $I\Delta$, $I\mu$ in α and χ reſpectively; and draw $\alpha\mu$.

Alſo draw $\chi\Psi$ parallel to $g\mu$, cutting Im in Ψ; and draw Ψa. Then is $I\Psi > Ic$, becauſe the Angles mec, $\mu\varepsilon\chi$ are equal as are alſo the Sides em, $\varepsilon\mu$; but the Angle $emc > \varepsilon\mu\chi$.

1 $kd : dl :: ea : ac$; and alſo $\kappa\omega : \omega x :: \varepsilon\alpha : \alpha\chi$.

2 $ea : ac ::$ Triang. eIa : Triang. aIc.

3 $\varepsilon\alpha : \alpha\chi ::$ Triang. $\varepsilon I\alpha$: Triang. $\alpha I\chi$

Tri-

4 Triang. $alc +$ Triang. $mac =$ Triang. $al\Psi +$ Triang. $\Psi am =$ Triang. ela.

5 Triang. $al\chi +$ Triang. $\chi a\mu =$ Triang. ela.

6 Triang. $al\chi$: Triang. $\chi a\mu :: l\chi : \chi\mu :: l\Psi : \Psi m ::$ Triang. $al\Psi$: Triang. Ψam.

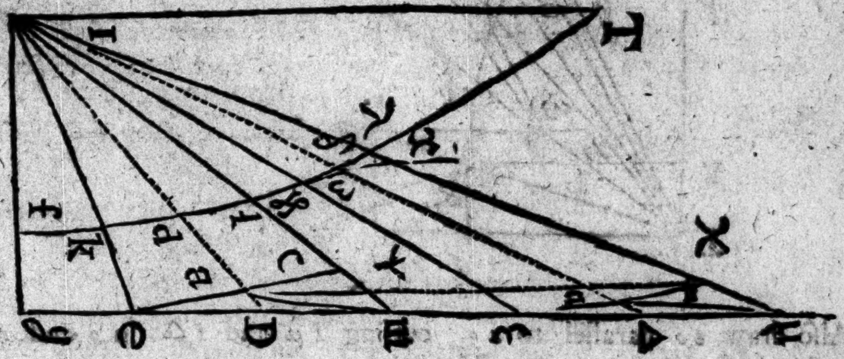
7 *Ergo* Triang. $\chi a\mu +$ Triang. $al\chi$ (Triang. ela) : Triang. $al\chi :: \varepsilon a : a\chi ::$ Triang. $al\Psi +$ Triang. Ψam (Triang. ela) : Triang. $al\Psi$.

8 *Ergo* $\varepsilon a : a\chi ::$ Triang. ela : Triang. $al\Psi :: \kappa\omega : \omega x$.

9 But $ea : ac ::$ Triang. ela : Triang. $alc :: kd : dl$.

10 And becauſe $l\Psi > lc$, *ergo* Triang. $al\Psi >$ Triang. alc, *ergo* Triang. ela : Triang. $alc >$ Triang. ela : Triang. $al\Psi$, *ergo*

11 $kd : dl > \kappa\omega : \omega x$; *ergo* much more $kd : dl > \kappa\delta : \delta\lambda$.



Lemma III.

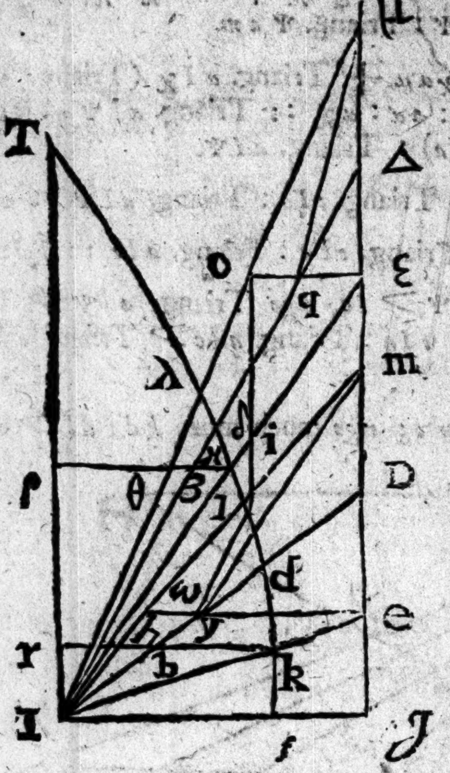
Conſtruct the Figure $TIfgeDm\varepsilon\triangle\mu$, and the Curve $fkl\kappa\lambda T$, drawing Ie, &c. as for the *Second Lemma.*

And draw kr, $\kappa\varrho$ parallel to Ig, cutting ID, Im, $I\triangle$, $I\mu$ in b, b, β, θ reſpectively.

Then is $kb : bb > \kappa\beta : \beta\theta$.

Demonſtration.

Draw ew parallel to kr, cutting Im, ID in w and y reſpectively; and draw ym.



Alſo draw εo parallel to $\kappa\varrho$, cutting $I\mu$ and $I\triangle$ in o and q reſpectively; and draw $q\mu$. Then is $\varepsilon o < ew$, becauſe the Angles wem, $o\varepsilon\mu$ are equal, as are alſo the Sides em, $\varepsilon\mu$; but the Angle $wme > o\mu\varepsilon$.

Draw oi parallel to $g\mu$, cutting Im in i. Then, becauſe $ew > \varepsilon o$, $Ii > Iw$, *ergo* Triang. $iyI >$ Triang. wyI.

1 | $kb : bb :: ey : yw$; and alſo $\kappa\beta : \beta\theta :: \varepsilon q : qo$.

2 | $ey : yw ::$ Triang. eIy : Triang. yIw.

3 | $\varepsilon q : qo ::$ Triang. εIq : Triang. qIo.

Tri-

4 Triang. $yIw +$ Triang. $myw =$ Triang. $yIi +$ Triang $myi =$ Triang. eIy.

5 Triang. $qIo +$ Triang. $\mu qo =$ Triang. eIq.

6 Triang. qIo : Triang. $\mu qo :: Io : \mu o :: Ii : mi ::$ Triang. yIi : Triang. myi.

7 *Ergo* Triang. $\mu qo +$ Triang. qIo (Triang. eIq) : Triang. $qIo :: eq : qo ::$ Triang. $yIi +$ Triang. myi (Triang. eIy) : Triang. yIi.

8 *Ergo* $eq : qo ::$ Triang. eIy : Triang. $yIi :: \kappa\beta : \beta\theta$.

9 But $ey : yw ::$ Triang. eIy : Triang. $yIw :: kb : bb$.

10 And becauſe the Triang. $yIi >$ Triang. yIw, *ergo* Triang. eIy : Triang. $yIw >$ Triang. eIy : Triang. yIi. *Ergo*

11 $kb : bb > \kappa\beta : \beta\theta$.

Scholium.

Firſt Paragraph. Let the Lines em, mF, Ff, SI, in *Fig.* 1. be in an Horizontal Plane; and let the Eye be at I.

Let alſo the Angles cmF, ISm, fFm be Right-angles; and let $eD = Dm$, and let $mP = PS$, and $mS = SF$.

Draw Ie, ID, Im, IP, and let the viſible Points at e, m, S be ſeen in k, l, s reſpectively.

Draw kl cutting ID in d, and draw ls cutting IP in p.

Then, by the *Scholium* of the *Seventh Problem*, eD will be ſeen as kd, Dm as dl, mP as lp, and PS as ps.

Draw kr parallel to mF, cutting Ie, ID, Im, IP, IS in k, b, b, n, r reſpectively.

Then if em and mS were to be put in Perſpective, and kr were a Section of the Perſpective Plane; kb would be the Perſpective of eD, and bb would be the Perſpective of Dm; alſo hn and nr would be the Perſpectives of mP and PS reſpectively.

Theſe things, and what follow in like Caſe, I only lay down, ſuppoſing my Reader to be acquainted with the Rules of Perſpective, of which many Books give Account; particularly *Lamy* hath compriſed the Rules hereof, in his Treatiſe of Perſpective, which is now extant in *Engliſh*.

Again, let the Lines $\varepsilon\mu$, $\mu\Phi$, $\Phi\varphi$, ΣJ, in *Fig.* 2. be in an Horizontal Plane; and let the Eye be at J; *viz.* let ΣJ be longer than $S\,l$, in *Fig.* 1. by the Length of $J\,l$; and let $\varepsilon\mu$, $\mu\Sigma$, *&c.* be

Fig. 1.

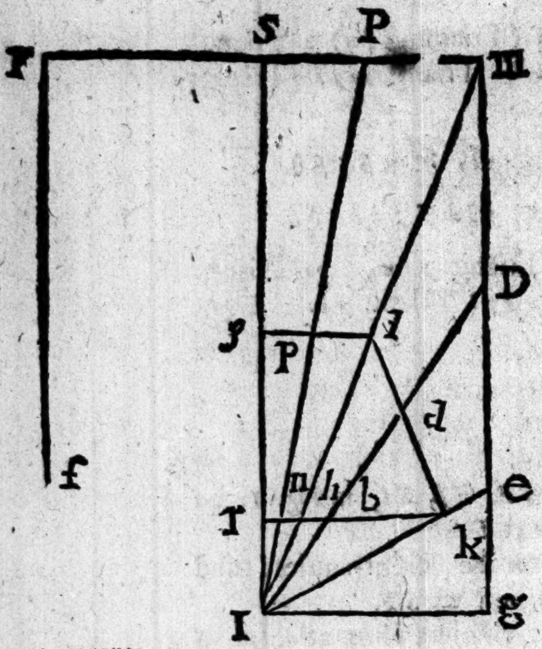
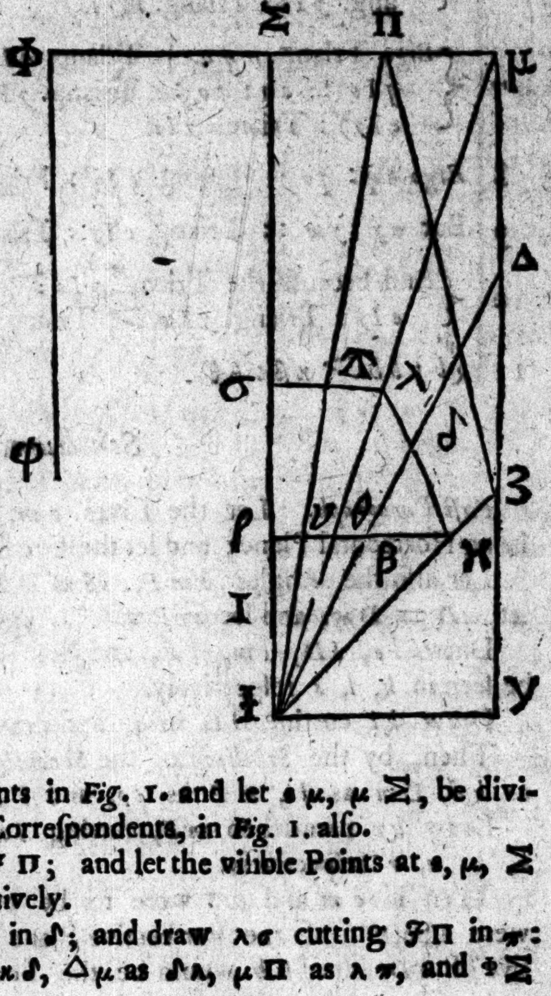


Fig. 2.



equal to their Correſpondents in *Fig.* 1. and let $\varepsilon\mu$, $\mu\Sigma$, be divided by Δ and Π as their Correſpondents, in *Fig.* 1. alſo.

Draw $J\varepsilon$, $J\Delta$, $J\mu$, $J\Pi$; and let the viſible Points at ε, μ, Σ be ſeen in κ, λ, σ reſpectively.

Draw $\kappa\lambda$ cutting $J\Delta$ in δ; and draw $\lambda\sigma$ cutting $J\Pi$ in π: Then $\varepsilon\Delta$ will be ſeen as $\kappa\delta$, $\Delta\mu$ as $\delta\lambda$, $\mu\Pi$ as $\lambda\pi$, and $\Phi\Sigma$ as $\pi\sigma$.

Draw $\kappa\varrho$ parallel to $\mu\Phi$, cutting $J\varepsilon$, $J\Delta$, $J\mu$, $J\Pi$, $J\Sigma$ in κ, β, θ, ν, ϱ reſpectively.

Then if $\varepsilon\mu$ and $\mu\Sigma$ were to be put in Perſpective, and $\kappa\varrho$ were a Section of the perſpective Plane; $\kappa\beta$ would be the Perſpective of $\varepsilon\Delta$, and $\beta\theta$ would be the Perſpective of $\Delta\mu$; alſo $\theta\nu$ and $\nu\varrho$ would be the Perſpectives of $\mu\Pi$ and $\Pi\Sigma$ reſpectively.

Second Paragraph. Now it is plain, that eD and Dm, which are ſeen as kd and dl, cannot be ſeen at the ſame Time as kb and bb upon Two Accounts; *firſt*, becauſe the Points d and l are far removed from their Correſpondents b and b; and, *ſecondly*, becauſe $kb : bb > kd : dl$. For if you were to make, as $kd : dl :: kb : kb$ to a 4th Proportional, that 4th Proportional would be longer than bb, becauſe it would be determined by the Interſection of kr, with a Line drawn from l parallel to db; and conſequently kb, the Perſpective of eD, in reſpect of bb, the Perſpective of Dm, would be diſproportionably bigger than eD appears in reſpect of Dm, when the Object it ſelf is expoſed to the Eye at the Diſtance and in the Situation given.

Again, For the like Reaſons it is evident, that $\epsilon\Delta$ and $\Delta\mu$, in *Fig.* 2. which are ſeen as $\kappa\delta$ and $\delta\lambda$, cannot be ſeen at the ſame time as $\kappa\beta$ and $\beta\theta$.

But yet it will appear, that $\epsilon\Delta$ and $\Delta\mu$, in *Fig.* 2. are more nearly ſeen as $\kappa\beta$ and $\beta\theta$, than eD and Dm, in *Fig.* 1. are ſeen as kb and bb; and that upon Two Accounts: *Firſt*, Becauſe θ is leſs removed from λ, than b is removed from l, as appears by *Lem.* 1. and conſequently the Points β and θ do more nearly come to the true Points δ and λ, than b and b do come to the true Points d and l.

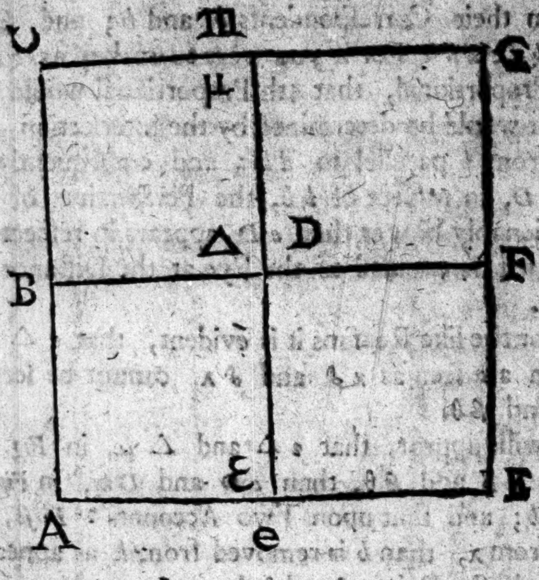
Secondly, Becauſe $kd : dl < \kappa\delta : \delta\lambda$, as appears by *Lem.* 2. And becauſe $kb : bb > \kappa\beta : \beta\theta$, conſequently the Proportion of $\kappa\beta$ to $\beta\theta$ is more nearly as $\kappa\delta$ to $\delta\lambda$, than the Proportion of kb to bb is as kd to dl; that is, $\kappa\beta$ and $\beta\theta$ are more nearly ſeen as $\epsilon\Delta$ and $\Delta\mu$ reſpectively, than kb and bb are ſeen as eD and Dm.

Corollary.

Let the Figure $EACG$ (*ſee the Figure following*) be a Parallelogram ſtanding perpendicularly upon the Ground-horizontal Line EG, and the Line em parallel to the Horizon, and of the Heighth of the Eye.

Let alſo the Line BDF, parallel to AE, divide the Parallelogram into Two equal Parts.

Then let Two Perſpectives of this Parallelogram be drawn, the Eye keeping at the ſame Vertical Diſtance Ig or $\zeta\gamma$, but placed upon the Horizontal Line of different Diſtances, *viz.* at *I* and *J*, as in the Figures for the *Scholium*, the Point *I* being very near to *r*.



Then will the Perſpective of the Parallelogram *A E F B*, when the Eye is placed at *I*, much more exceed in Breadth the Perſpective of the Parallelogram *B F G C*, than the Parallelogram *A E F B* it ſelf will ſeem to exceed *B F G C* when beheld from the Point *I*.

But when the Eye is conſiderably removed to *J*, the Perſpectives and the real Appearances more nearly reſemble one another in Breadth, becauſe the Breadths of the Perſpectives of the Parallelograms are the Perſpectives of *e D* and *D m*, as thoſe are eaſily ſenſible of who know Perſpective.

And for the farther Proof of this Concluſion I appeal to Experience.

Third Paragraph. Now although the farther *J* be removed from Σ, on the Line Σ *I*, the nearer will the Perſpectives of the Parallelograms *A E F B*, *B F G C*, approach to the true appearance of theſe Parallelograms to the Eye it ſelf, yet *J*, can never be removed ſo far as to make the Perſpectives and the Natural Appearances of thoſe Parallelograms to the Eye exactly proportional to one another, becauſe $\kappa\delta : \delta\lambda$ will always be $\leq \kappa\beta : \beta\theta$, as might eaſily be ſhewn. Yet

κβ

$\kappa\beta$, may be ſo nearly to $\beta\theta$, as $\kappa\delta$ to $\delta\lambda$, as that the Difference may be but little ſenſible to the Eye, which diſtinguiſhes not over-nicely in this Caſe. And the beſt Piece of Perſpective we can have of Figures, which have Lines of any conſiderable Length perpendicular to the Perſpective Plan, muſt have the Eye placed at ſuch a Diſtance as to make the Difference between $\frac{\kappa\delta}{\delta\lambda}$ and $\frac{\kappa\beta}{\beta\theta}$ ſo ſmall, as not to be heeded by the Eye; that is to ſay, If you were to make as $\kappa\delta : \delta\lambda :: \kappa\beta :$ to a 4th Proportional, that 4th Proportional muſt be ſo little bigger than $\beta\theta$, as that the Difference between it and $\beta\theta$ would be but little regarded by the Eye.

Call $\left\{\begin{array}{l}\kappa\delta = m\\ \delta\lambda = n\\ \kappa\beta = p\\ \beta\theta = q\end{array}\right\}$ Then is the 4th Proportional $= \frac{np}{m}$. call the biggeſt Difference which the Eye will bear between q and $\frac{np}{m} = \frac{q}{x}$.

Note, I aſſume the biggeſt Difference which the Eye will bear between q and $\frac{np}{m}$, becauſe I make uſe of this Quantity to find a fixed Rule for determining the Diſtance the Eye muſt be at from Σ, to make the Perſpectives of the Parallelograms A E F B, B F G C, correſpond with the natural Appearance of them to the Eye: And ſuch Rule ought to admit the Eye as near as poſſible to Σ, becauſe many Objects are ſeen when the Eye is near, which are obſcured and hid by interpoſing Bodies when the Eye is farther off, and for other Reaſons too. And the bigger the Difference is between q and $\frac{np}{m}$, the nearer the Eye is admitted, as may be gathered from the Second and Third *Lemma's* foregoing.

Problem XII.

Let the ſeveral Denominations foregoing ſtand. Then let it be required to find x.

Take ſome plain upright Wall, and draw upon it Two equal Rectangles A E F B, B F G C, as in *Fig.* for *Corol.* of *Second Paragraph*; and draw the Horizontal Line $\epsilon\,\Delta\,\mu$, of the Heighth of your Eye, ſo you have $\epsilon\,\Delta$, and $\Delta\,\mu$, equal to each other. Then taking the

Point Σ, at any Diſtance without the Wall, and of the Horizontal Hejghth of the Eye; as in the 2*d Fig.* for the *Scholium*, draw $\Sigma\, \mathcal{J}$, a long Line parallel to $\iota\, \Delta\, \mu$. Then taking ſeveral Points in the Line $\Sigma\, \mathcal{J}$, draw ſo many ſeveral Perſpectives of the Parallelograms *A E F B*, *B F G C*; and then comparing each Perſpective with the natural Appearance of the Parallelograms at each Point where the Eye is placed in the Perſpective Draught, the neareſt Perſpective which will ſuit with the natural Appearance muſt be pitched upon to calculate from.

This being pitched on, you have $\mathcal{J}\, \iota$, $\mathcal{J}\, \mu$, and the Triangle $\iota\, \mathcal{J}\, \mu$, which may very commodiouſly be contracted by a Scale. And then by the 2*d Problem* finding $\mathcal{J}\, \kappa$, $\mathcal{J}\, \lambda$, namely the Diſtances at which the viſible Points at ι and μ are ſeen, and drawing $\kappa\, \lambda$ and $\mathcal{J}\, \Delta$, you have $\kappa\, \delta$ and $\delta\, \lambda$. And then drawing $\kappa\, \varsigma$ at right Angles to $\iota\, \mu$, you have $\kappa\, \beta$ and $\beta\, \theta$. That is, ſuiting to the Denominations above, you have m, n, p, q, and conſequently $\frac{np}{m}$.

Then $\frac{q}{x}$ being the Difference between q, and $\frac{np}{m}$, as before;

$\frac{np}{m} = q + \frac{q}{x}$. Ergo $xnp - xqm = mq$.

Ergo $x = \frac{mq}{np - qm}$.

By this Method, any one who pleaſes may proceed for the finding of x. And by the ſame I proceeded; and calculating according to my Apprehenſion, I found $x = 5$ nearly.

Coralleries.

1. Hence the biggeſt Difference which the Eye will bear between q, and $\frac{np}{m}$, is $\frac{q}{5}$, or $\frac{1}{5}$ of q.

And it may be conſidered, that the Eye the rather bears ſo great a Difference; becauſe, let $\kappa\, \delta$ be to $\delta\, \lambda$ in what Proportion it will, by this Method, which only conſiders the local Length of them, $\kappa\, \delta$ will really appear bigger in Proportion to $\delta\, \lambda$; becauſe $\delta\, \lambda$ being more oblique to the Eye than $\kappa\, \delta$, the meaſured Length of $\delta\, \lambda$ will fall more ſhort of the local Length of it, than the meaſured Length of $\kappa\, \delta$ falls ſhort of the local Length of it; as is plain by conſidering the manner of applying the Segmental in the *1ſt Problem.* And ſo the Proportion which $\kappa\,\delta$ appears to have to $\delta\,\lambda$, will ſtill come nearer to the Proportion of $\kappa\,\beta$ to $\beta\,\theta$.

2. Hence ariſeth a Rule for finding the neareſt commodious Diſtance for placing the Eye upon the Line $\Sigma\,\zeta$, in the Art of Perſpective; namely, this which follows:

A Rule for finding the neareſt commodious Diſtance for placing the Eye in the Art of Perſpective.

Let the Line $\epsilon\,\mu$ be the Horizontal Section, at the Heighth of your Eye, of your Plane *AEGC*, which is perpendicular to the Picture or Perſpective Plan; (ſee the Figures of the *Scholium* and of the *Corollary* of the *2d Paragraph*) and let $\mu\,\Sigma$, or $\zeta\,\gamma$, be the Vertical Diſtance of $\Sigma\,\zeta$, parallel to $\epsilon\,\mu$, and at the Heighth of your Eye.

Take any Point in the Line $\Sigma\,\zeta$ for ζ, (the farther off from Σ, the better) and drawing $\zeta\,\epsilon$, $\zeta\,\mu$, take the Length of each; and by the *2d Prablem* find out $\zeta\,\kappa$, $\zeta\,\lambda$, namely the Diſtances at which the Viſible Points at ϵ and μ are ſeen; and drawing $\zeta\,\triangle$, biſecting $\epsilon\,\mu$ in $\triangle$, draw next $\kappa\,\lambda$, cutting $\zeta\,\triangle$ in δ; ſo you have $\kappa\,\delta$, and $\delta\,\lambda$. Then draw $\kappa\,\varsigma$ perpendicular to $\epsilon\,\mu$, cutting $\zeta\,\triangle$, $\zeta\,\mu$, in β and θ, reſpectively; ſo you have $\kappa\,\beta$, and $\beta\,\theta$. Then ſuiting to the Denominations at the End of the *3d Paragraph*: If $\beta\,\theta + \frac{1}{3}$ of $\beta\,\theta$ be $=$ or $> \frac{n\,p}{m}$, then is ζ placed far enough from Σ. But if $\beta\,\theta + \frac{1}{3}$ of $\beta\,\theta$ be $< \frac{n\,p}{m}$, then ζ muſt be placed farther off. And ſo by ſeveral Trials you will gain your Purpoſe.

The Reaſon of this is obvious from the Conſideration of the Third Paragraph of the Scholium, and of the 12th Problem.

Note, *The longer $\mu\,\Sigma$ is, the nearer may* J *be admitted to make*

$$\beta\,\theta + \tfrac{1}{3}\ \text{of}\ \beta\,\theta = \frac{n\,p}{m}$$ *as might eaſily be* [illegible] shewn.

And thus you have a Rule fully ſufficient for finding the neareſt commodious Diſtance of the Eye from the Object in the Art of Perſpective, which is applicable to all Caſes, except where the Diſtance is determined, where the Length of $\epsilon\,\mu$ is ſo great, and the Length of Σ μ ſo little, as to cauſe *J* to fall at an immenſe Diſtance, or where the Figure of the Object is ſuch, as that it can only be ſeen in its true Form from a Point at a determinate Diſtance. In theſe Caſes, where the Point is near, there can be no good Perſpective Draught made.

But that otherwiſe 'tis a ſufficient Rule, it may appear eaſily.

For Inſtance: if Φ φ, φ μ, were horizontal Sections at the Heighth of the Eye of the Two oppoſite parallel Walls, adorn'd with Windows, &c. and Φ μ, an horizontal Section at the ſame Heighth of another Wall, joining the other Two at Right Angles; in ſuch Caſe, the Point Σ being taken from the Verticle Point, either in the midſt of φ μ, or otherwiſe as is moſt commodious; the leaſt Diviſion of φ μ, muſt be taken for Σ μ, and the Diſtance of *J* muſt be found by the Rule in the Second Corollary of the Twelfth Problem; becauſe the ſhorter Σ μ is, the longer muſt be the length of Σ *J*, as by the Note above appears. And therefore if the Perſpectives of φ Δ, Δ μ, will agree with the natural Appearance of them from the Point *J*; much more will the Perſpectives of the Parts of Φ φ, agree with the natural Appearance of them.

And ſo there is no need to find the Diſtance *J*, to give agreeable Perſpectives of the Parts of φ Φ, or of the Parts of any ſloping Line φ Π in *Fig.* 2. And much leſs is there need of finding *J*, to give agreeable Perſpectives of the Parts of Φ μ, becauſe θ ν, ν ϛ, the Perſpectives of μ Π, Π Σ, will always be very nearly proportional to λ π, π σ, the Two Appearances of them; and beſides θ ϛ is very nearly ſituated, as λ σ is alſo.

And by this Rule, in many Caſes, Painters, Limners, &c. may either take ſuch Diſtances of *J* from Σ, as will beſt anſwer their Purpoſe; or if their Diſtance be limited, they may enlarge μ Σ to comply with it; or if both be limited, they may ſhorten φ μ to comply with both; (for the ſhorter φ μ is, the nearer may *J* be admitted to comply with this Rule, as might be made appear). But if all Three be limited, and the Diſtance be too near to comply with the Rule given; then, as I ſaid before, there can be no good Perſpective of ſuch Object.

FINIS.

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42